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SUMMARY

Data from transverse line sampling of continous plumes over a water surface indicate that the concentration distributions at fixed locations, r, relative to the centre of gravity location, r=0, is reasonably well described by the probability of vanishing concentration, F(0,r), and a log-normal distribution for non-zero concentrations with parameters $\chi_o(r)$ and $\sigma_*(r)$. F(0,r) is estimated to be small and approximately constant in the interior of the mean cloud and increase rapidly for larger r-values. $\chi_o(r)$ is estimated to have a nearly Gaussian shape. $\sigma_*(r)$ is estimated to increase from approximately 1 at r=0 to 1.5 at the boundaries of the mean cloud. The transverse joint properties of the concentration fluctuation field are dominated by large scale variations.

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1 INTRODUCTION

Numerous articles describe how the mean size of clouds increases with time in a turbulent flow. Still, there are flows and/or clouds for which there are large uncertainties concerning this point (Hanna et al, 1). In comparison, there have been very few articles on how to describe other aspects of the stochastic concentration field of clouds (Csanady 2,3; Kornreich 4).

We will use data from transverse line sampling of passive scalar plumes in flows over a sea surface to estimate properties of the concentration field relative to the location of centre of gravity. The properties are concentration distributions at fixed locations and simple aspects of transverse joint properties. Our results on concentration fluctuations, as others, should be considered as tentative only.

2 DATA

2.1 Experiments

The ground level source was steady and continuous. At a downwind distance of $x \simeq 500$ m, sampling was done from a boat crossing the cloud at a speed of approximately 3 ms^{-1}. It is assumed that the time used for cloud crossing is unimportant for the statistical properties we are discussing. The transverse resolution and averaging distance was estimated as $\Delta y \simeq 3.5$ m. In each experiment there were 12 cloud traverses. A total of 16 experiments were made. The traverses with only zero readings are excluded, because the centre of gravity location cannot be defined. This also applies to traverses with less than 4 concentration readings near the end of the sampling line. Of a total of N = 192 crossings there were only ~~approximately~~ N ≃ 100 for which the sampling line was reasonably certain to cover the cloud.

The atmospheric flows may be characterized as near neutral flows over a water surface. The mean wind varied between 2 ms^{-1} and 7 ms^{-1}.

2.2 Data transformation

It is only possible to estimate the centre of gravity for the sampling line. As some of the instantaneous cloud was sometimes located outside the sampling line, the centre of gravity location is estimated by fitting a Gaussian curve to the observations. When $\chi(y)$ is the crosswind concentration profile, the regression model is:

$$\hat{\chi}(y) = Q \exp\left[-\frac{1}{2\sigma^2}(y-\mu)^2\right] \qquad (2.1)$$

Q is representative of the maximum magnitude of the concentration profile. μ is the center of gravity, and σ is a measure of cloud size. In $(\ell n\chi(y), y)$ coordinates, the regression curve is a second-order polynomial, and is fitted to data as described by Forsythe (5). Only data points in the interior of the cloud are included in the analysis. If a zero concentration reading occurs there, it is replaced by a small value so that infinite logarithms are avoided. If the instantaneous concentration profile consists of two distinct clouds, so that the coefficient before the quadratic term becomes negative or assumes very small positive values, the centre of gravity is estimated in the conventional way.

It turns out that the choice of data to be included in the regression affects the estimated coefficients $\{Q,\mu,\sigma\}$ significantly. However, it appears that the estimated mean value of Q for one experiment varies less with reasonable selection choices than typical $\bar{\chi}_{max}$ - variation from experiment to experiment.

To minimize the systematic (weather) variations from one experiment to another, the concentration readings for one experiment are normalized to the estimated mean value of Q for the experiment. The subjective, or arbitrary nature of this transformation is obvious but inevitable. The normalized concentration profiles oriented relative to their centre of gravity, are assumed to be realizations from the same distribution, representative for a near neutrally stratified flow over a water surface.

3 ANALYSIS

3.1 Concentration distribution at fixed locations

The sample size $N \simeq 100$ is too small for accurate estimates of distribution functions. It is increased somewhat by assuming the density $\beta(\chi;r)$, $r = y-\mu$ to be symmetrical with respect to the centre of gravity, $r = 0$, so that the samples at r and $-r$ can be considered to originate from the same population. However, as the concentrations at r and $-r$ are stochastically dependent, the increase is not equivalent to a factor of two.

The probability of zero concentration (intermittency factor, $F(0,r)$) is difficult to estimate because there were a significant number of crossings with either zero readings only and/or the cloud almost missed the sampling line. Based on the normalized concentration profiles, arranged as a large matrix, we estimate $F(o,r)$ approximately as indicated in Figure 3.1. The intermittency factor is estimated to be small and fairly constant in the interior of the cloud ($r \leq \bar{\sigma}$), and to increase rapidly in the interval $\bar{\sigma} < r < 3\,\bar{\sigma}$.

The estimated distribution of non-zero concentrations is given in Figure 3.2. It is definitely skewed to the right and may be represented reasonably well by log-normal distribution:

$$\beta(\chi) \simeq \frac{1}{\sqrt{2\pi}\sigma_* \chi} \exp\{-\frac{1}{2\sigma_*^2} [\ln\chi - \ln\chi_o]\} \qquad (3.1)$$

The distribution may then be discussed in terms of the parameters χ_o and σ_*, which are related to the more easily visualizized first and second central moments, $\bar{\chi}$, and $\overline{\chi'^2}$, as:

$$\chi_o = \bar{\chi}\,[1 + \overline{\chi'^2}/\bar{\chi}^2] \tag{3.2}$$

$$\sigma_* = \{\ln\,(1 + \overline{\chi'^2}/\bar{\chi}^2)\}^{\frac{1}{2}} \tag{3.3}$$

The parameters χ_o and σ_* are estimated from the data as:

$$\widehat{\ell n \chi_o} = \frac{1}{N}\sum_{i=1}^{N} (\ell n \chi)_i \tag{3.4}$$

$$\widehat{\sigma^2_*} = \frac{1}{N}\sum_{i=1}^{N} [(\ell n \chi)_i - \widehat{\ell n \chi_o}]^2 \tag{3.5}$$

Figure 3.3 shows that the estimated spatial variation of $\chi_o(r)$ or $\bar{\chi}(r)$ resemble Gaussian functions, as they should. The concentration fluctuation parameter σ_* is estimated to increase from approximately 1 in the interior of the cloud to approximately 1.5 near the boundaries. The estimated σ_* is larger than the value of 0.2 suggested for flows over a sea surface by Csanady (2). It is also larger than the σ_* values for hourly dosages as estimated by Eidsvik and Hansen (6). A reason for our large σ_* could be that systematic weather variations have not been properly removed by the normalization procedure. However, the presented $\hat{\sigma}_*$ is the minimum with respect to all normalization procedures tried.

3.2 Transverse joint properties

The statistical properties of $\chi(r)$ are described when probabilities can be assigned different functional forms. We will discuss this complicated subject in terms of empirical orthogonal functions. Included in the analysis are only realizations for which the sampling line most probably covered the cloud. The estimated mean value and standard deviation for this sample are shown in Figure 3.4. The $\bar{\chi}$ compares very well with the Gaussian spatial distribution of mean concentrations. When the relation (3.3) is used, it is observed that σ_* is estimated as approximately 0.8 for the interior of the cloud. This compares reasonably well with Figure 3.3. The fluctuations $\chi'(r)$ are the variables to be analyzed in terms of empirical orthogonal functions.

As discussed by, for instance, Eidsvik (7) an empirical orthogonal function $k(r)$ bears the closest resemblance to the ensemble of functions $\chi'(r)$. With the covariance matrix $Q(r,r') = E\chi'(r)\chi'(r')$ the functions $k(r)$ are found as solutions to the eigenvalue problem:

$$\int_{-\infty}^{\infty} Q(r,r')k(r')dr' = \lambda k(r) \qquad (3.5)$$

Or with discrete data:

$$\sum_{j=1}^{N} Q(r_i,r_j)k(r_j) = \lambda k(r_i) \qquad (3.6)$$

It may be shown that:

$$\chi'(r_j) = \sum_{i=1}^{N} c_i k_i(r_j) \qquad (3.7)$$

with coefficient variance:

$$E\, c_i c_j^* = \lambda_i \delta_{ij} \qquad (3.8)$$

If λ_i decreases fast enough with i, the most important aspects of the stochastic $\chi'(r)$ - function are therefore contained in a few stochastic coefficients c_i, with variance as given by Equation 3.8. The estimated eigenvalues λ_i, shown in Figure 3.5, indicate that $\chi'(r)$ may be represented well with a few empirical orthogonal functions. For scales that are small relative to the cloud, the $\chi'(r)$-field must approximately be locally homogeneous. The "best" representation is then, as shown by Lumley (8), a Fourier representation. The λ_i-values will therefore approach the power spectrum of a passive scalar in homogeneous turbulence, as i becomes large. In the inertial subrange this spectrum is predicted to obey a -5/3 law (Corrsin 9; Batchelor 10; Gibson and Schwarz 11, and Tennekes 12). The fact, that our λ_i-values decrease faster with decreasing scale, is likely to be caused by a larger averaging distance than Δy = 3.5 m.

The three first empirical orthogonal functions are shown in Figure 8. It is observed that the first and most important is associated with the largest scale of the $\chi'(r)$-field and so that, when $\chi'(r)$ is large, it is largest at the cloud centre. The second most important aspect of the $\chi'(r)$ variation is that when the concentration is high at one side of the cloud, it is small at the other. The third most important aspect is that when the concentration is high at the cloud center, it is small at the boundaries. The higher order, less important orthogonal functions are of smaller and smaller scales.

It is obvious that the normalization procedure, or systematic weather variations from experiment to experiment, could contribute to the dominance of large scale concentration fluctuations. The above results may therefore not be correct in detail. However, compared to other experimental evidence of the dominance of large scale variations in the atmosphere, the results can be regarded as qualitatively real.

4 CONCLUDING REMARKS

The concentration distributions at fixed locations relative to the centre of gravity may be reasonably well described by the probability of vanishing concentration F(o;r) and a log normal distribution of non-zero concentrations, with parameters $\chi_o(r)$ and $\sigma_*(r)$. F(o;r) is estimated to be reasonably small and constant in the interior of the mean cloud and to increase rapidly near the boundaries. The mean cloud, as described by $\chi_o(r)$, is approximately Gaussian. The fluctuations, as described by $\sigma_*(r)$, are larger than estimated by others and increase with the distance from the centre of gravity.

The estimated transverse joint properties of the concentration field indicate that most of the variations are associated with the largest scales.

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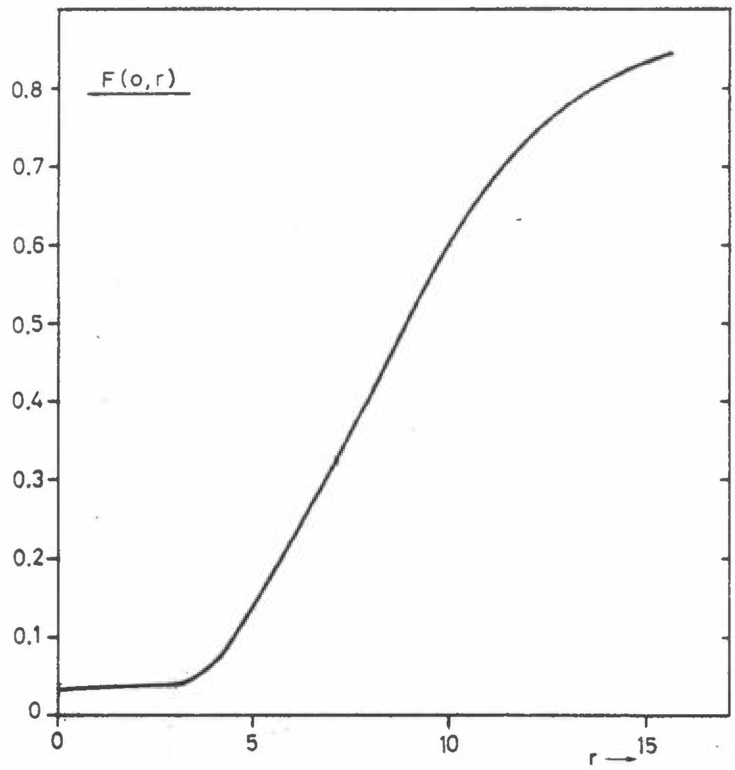

Figure 3.1: *Estimated probability of vanishing concentration as function of transverse distance from the cloud's centre of gravity.*

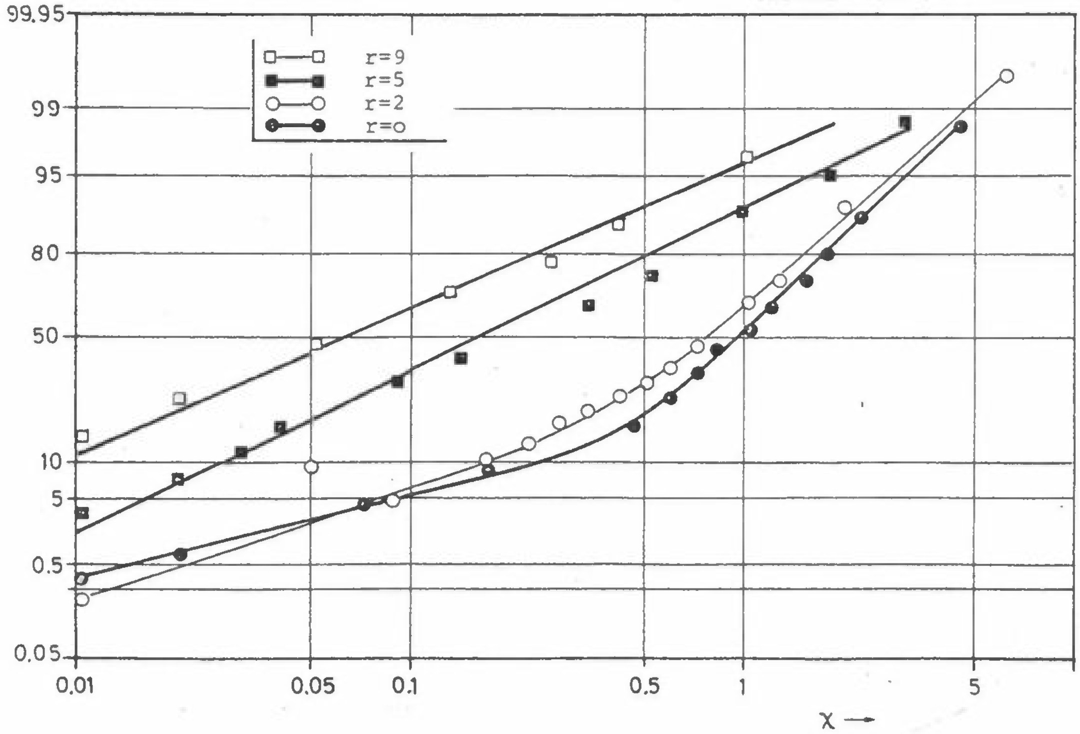

Figure 3.2: *Estimated cumulative distribution of non-zero concentrations at different locations relative to the cloud's centre of gravity. (Log-Gaussian coordinates.)*

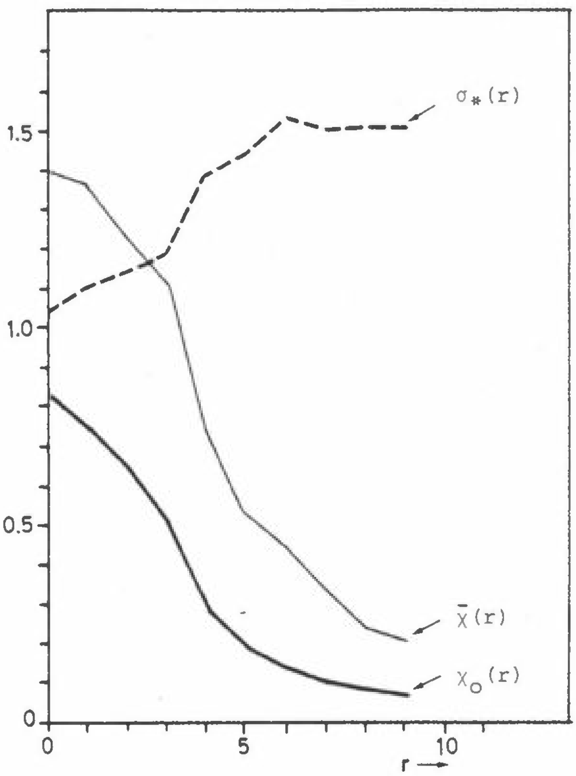

Figure 3.3: *Estimated parameters for the log-normal distributions at different locations relative to the cloud's centre of gravity. (Normalized data.)*

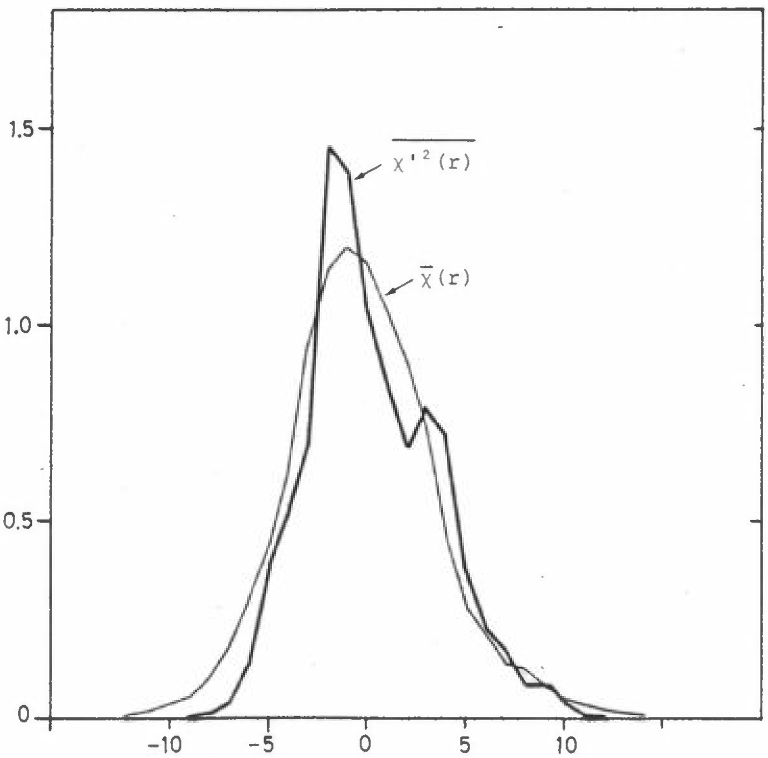

Figure 3.4: *Estimated mean and variance at different locations relative to the cloud's centre of gravity. (Only the "best", normalized data included.)*

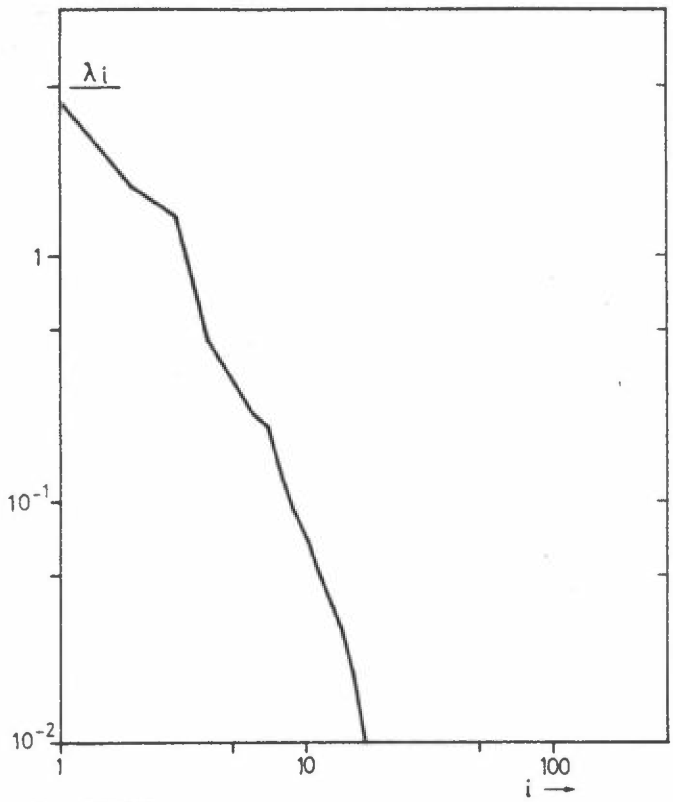

Figure 3.5: *Eigenvalues in empirical orthogonal representation arranged according to magnitude. (Normalized data.)*

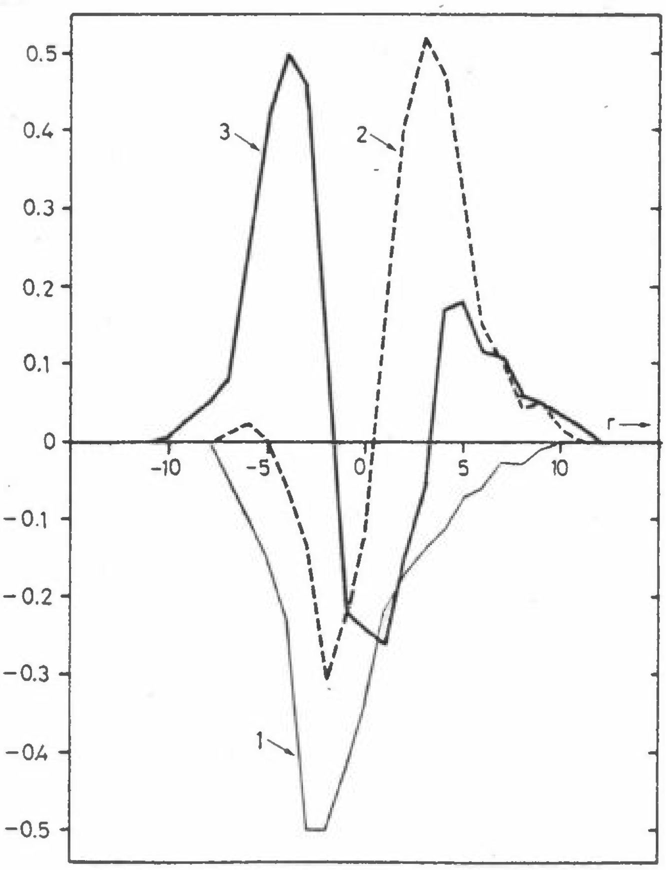

Figure 3.6: *The three empirical orthogonal functions associated with the largest eigenvalues.*

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OPPDRAGSGIVER Oljedirektoratet		
3 STIKKORD (á maks.20 anslag) Diffusion		

REFERAT (maks. 300 anslag, 5-10 linjer)

Målinger av instantan konsentrasjon langs linjer på tvers av en kontinuerlig fane er blitt benyttet til å estimere statistiske egenskaper ved konsentrasjonsfeltet.

TITTEL Estimates of concentration fluctuations in an instantaneous cloud

ABSTRACT (max. 300 characters, 5-10 lines)

Data from transverse line sampling of continous plumes is used to estimate the stochastic structure of the concentration field.

**Kategorier: Åpen - kan bestilles fra NILU A
Må bestilles gjennom oppdragsgiver B
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