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MESOSCALE WIND FIELD ANALYSIS
A SURVEY OF RECENT LITERATURE

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MESOSCALE WIND FIELD ANALYSIS, A SURVEY OF RECENT LITERATURE

1 INTRODUCTION

One of the most important requirements in mesoscale air quality modeling is an accurately specified mean wind field over flat or complex terrain. Analytical solutions for three-dimensional stationary flows in the atmospheric boundary layer over terrains may be obtained (Kao, 1980), however in most practical cases a numerical solution of the full Navier-Stokes equations is not a feasible way of constructing mass consistant wind fields.

As a result, simpler objective analysis procedures are used. The most common approach for generating a discrete wind field consists of a two-step procedure. In the first step an initial field has to be established, the next step is to employ an objective analysis procedure to adjust the wind vectors of each grid point so that appropriate physical constraints are satisfied (Goodin et al, 1980). Another possible solution is to determine a stream function from the observed wind data.

In populated areas the following data can usually be obtained (Mc Rae et al., 1982): synoptic meteorological charts, geostrophic winds (Ug), terrain height (h), surface roughness (z_o), temperature (T), relative humidity, surface winds and estimates at 850, 700, 500 mb heights. The availability of this information is an important factor in selecting the procedures for applications.

The general approaches used for wind field generation are shown in Figure 1. In the following different methods will be presented.

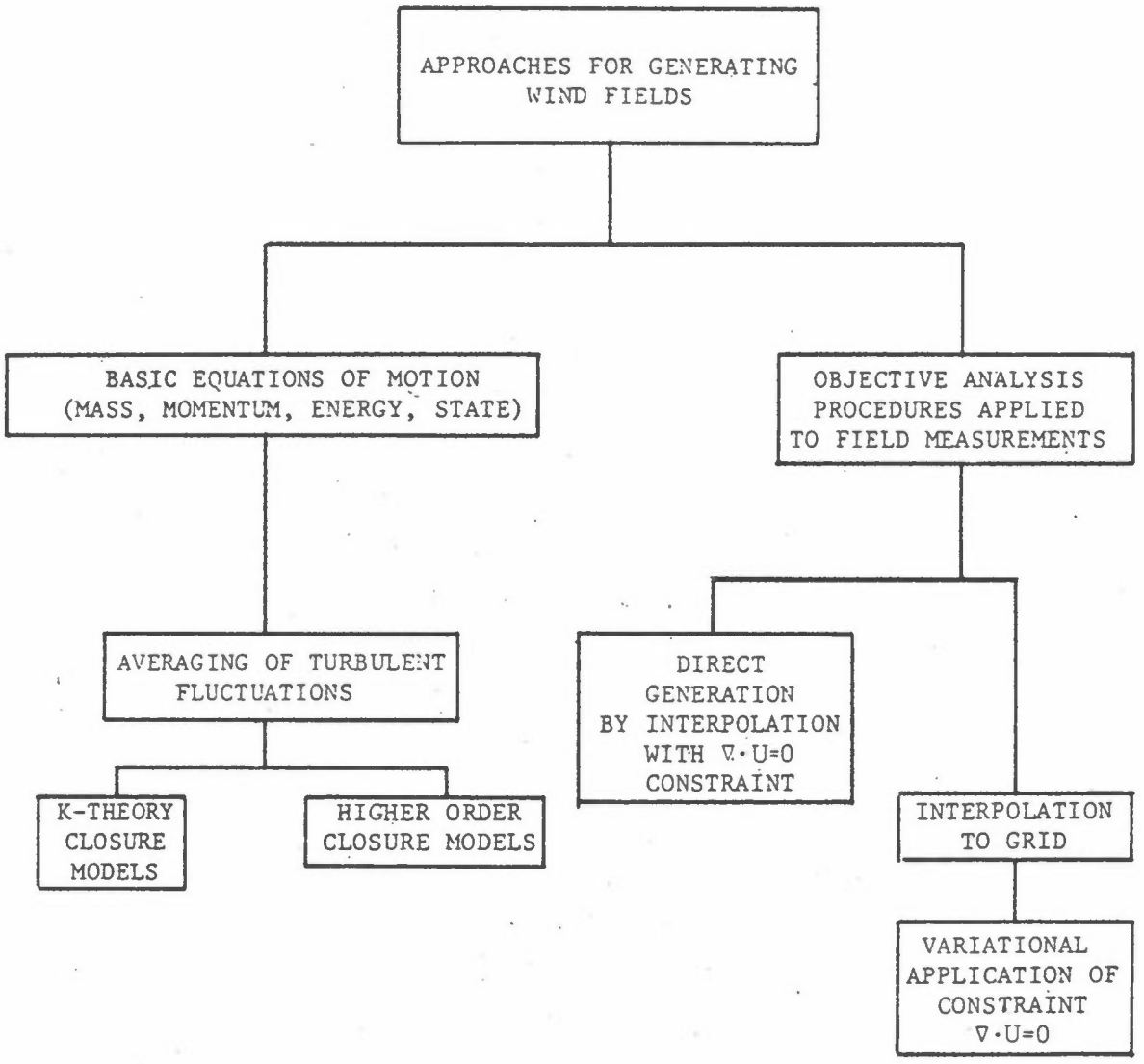

Figure 1: Summary of approaches for generating wind fields. (After McRae et al, 1982).

2 BASIC EQUATIONS

The mean state of the atmosphere is defined by the following meteorological variables (Drake, 1980):

- Velocity field, U[u,v,w]
- Density field, ρ
- Pressure field, p
- Temperature field, T (2.1)
- Phases of water, r_i
- Solar radiation as a function of band widths, R_i
- Correlations between mean fields, c_{ij}

These quantities are governed by a following set of Reynolds-averaged equations of motion:

- thermodynamic equation
- momentum equation (2.2)
- conservation of mass (continuity) equation
- equation of state

These equations provide constraints, which should be satisfied for the wind field at each grid point. Since measurements of the variables described by (2.1) are rarely available, continuity equation is chosen as only one constraint in most practical applications. In this way the mass-consistant wind fields are generated.

The continuity equation can be written as:

$$\frac{1}{\rho}\frac{d\rho}{dt} + \text{div}\ \bar{U} = 0 \qquad (2.3)$$

where:

$$\text{div}\ \bar{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Two simplifications of the continuity equation (2.3) can be presented (Drake, 1980):

- "anelastic" approximation of Ogura and Phillips (1962), which provides the following form of the continuity equation:

$$\frac{w}{\rho}\frac{\partial \rho}{\partial z} + \text{div}\ \bar{U} = 0 \qquad (2.4)$$

where $\rho = \rho(z)$. The flow is incompressible and density-stratified.

- when the depth of the domain $\bar{h}$ is less than 1 to 3 km, $\rho(z)$ can be assumed to be constant. Thus, the flow is incompressible and nonstratified. Continuity equation takes the Boussinesq form:

$$\text{div}\ \bar{U} = 0 \qquad (2.5)$$

In a mountainous terrain it may be necessary to consider (2.4), in other cases (2.5) can often be applied.

3 THE DIRECT METHOD OF TWO-DIMENSIONAL WIND FIELD GENERATION

From the well-known Helmholtz theory, separation into non-divergent and irrotational parts can be made for any vector field that is continuous and piecewise differential. The horizontal wind field ($\bar{V}$) may be represented as the sum of two components:

$$\bar{V} = \bar{V}_{\psi} + \bar{V}_{\chi} \qquad (3.1)$$

where:

$\bar{V}_\psi = k \times \nabla_h \psi$ is nondivergent

$\bar{V}_\chi = \nabla_h \chi$ is irrotational

ψ - stream function

χ - velocity potential

$\nabla_h = [\frac{\partial}{\partial x}, \frac{\partial}{\partial y}]$

The relative vorticity (ζ) and divergence (D) of $\bar{V}$, are then given by the two Poisson equations:

$$\zeta = \nabla_h^2 \psi \qquad (3.2)$$
$$D = \nabla_h^2 \chi$$

Using the relationships between stream function and vorticity or between velocity potential and divergence, the wind field may be estimated, when D and ζ are known. Such methods are, however, dependent on boundary conditions, which are difficult to describe (usually boundaries are placed in data-sparse areas).

This type of method was used by Grønskei (1973) to estimate the wind field over Oslo. The following set of equations were used:

Within the region:

$$D = \nabla_h^2 \chi = a \cdot f(T) \qquad (3.3)$$

f(T) represents an empirically estimated function which depends on the temperature differences between two height levels in the area

$$\zeta = \nabla_h^2 \psi = 0 \qquad (3.4)$$

and on the boundary:

$$\chi = 0 \qquad (3.5)$$

ψ was estimated from observed winds

4 THE TWO-STEP PROCEDURE TO GENERATE WIND-FIELDS

As a starting point the boundaries of the region, the vertical extention and basic grid cell size must be chosen. Once the grid has been established, surface wind velocity measurement (for 2D and 3D approaches) and upper level wind data (for 3D approaches) are interpolated to specify initial values for each computational point (Goodin et al., 1980). The final step is to adjust the velocity field so that applied physical constraints are satisfied.

4.1 Initial wind field

The methods used to construct the initial wind field are different for 2- and 3-dimensional cases.

- Two-dimensional (2D) approaches.

In this case a surface mean wind has to be established as a starting point for the objective analysis. The air is assumed to be well mixed within the chosen layer, and the computed surface wind is treated as a mean wind in this layer. (The vertical component w_o is usually set to zero).

The most common method for obtaining the initial surface wind field is by simple interpolation of the station data to a finer mesh. However, more refined methods may also be used. They are:

- the method described in section 3.

- spectral method - where a family of surfaces of a periodic functional form (sine and cosine waves for Fourier analysis) is spectrally weighted to provide a desired degree of approximation to the data points.

- initial wind field generation by assuming a balance between Coriolis, pressure gradient and frictional forces. (Method used by Danard, 1977 and McEwen et al., 1980).

- Three-dimensional (3D) approaches.

First an initial surface wind field has to be computed. Since measurements of w_o rarely are available, this value is assumed to be zero at each grid point. In the next step the vertical variations of the horizontal winds above each grid point are specified. The following procedure can be applied:

a) The standard power law is used up to the height of the surface layer.

b) Measured vertical profiles are used from there to the top of the grid or (if not available) a linear variation is assumed for the wind between the surface layer and the upper boundary (taken from the synoptic analysis).

When measured upper air data are available, the horizontal velocities at each level can also be obtained from an interpolation of these data. The resulting field is used as the initial 3D wind field.

In addition to this it can be mentioned that Trombetti and Tampieri (1983) recently developed a numerical method for objective interpolation of data in the height-time domain, when the time intervals of measurements are irregular, and the heights of the radio soundings are very different. However, this case will not be considered here.

4.2 Methods for interpolation of sparse data

The problem can be expressed as follows: given a bounded, two-dimensional region, containing n error-free data with values C_k at locations $\mathbf{x}^k = [x,y]$, $k = 1, \ldots .n$, develop a function $f(\mathbf{x})$, which assigns a value C to any arbitrary location $\mathbf{x}$. The assumption is made, that measurements are given at roughly regular time intervals.

a. The weighted interpolation method

In this method the assumption is made, that the grid value is some

weighted average of the surrounding data values, (Goodin et al., 1979).

$$C_{ij} = \sum_{k=1}^{n} C_k W_k(r) / \sum_{k=1}^{n} W_k(r) \qquad (4.1)$$

where:

C_k = measured value at the kth measuring station
$W_k(r)$ = weighting function
r = distance from the grid point to the station

Various forms of the weighting function W(r) have been used. At some distance, R, called "radius of influence" the weighting function goes to zero.

$$\lim_{r \to R} W(r) \to 0 \qquad (4.2)$$

That means that each measurement station has a fixed (or variable) "area of influence" (AF) among the grid points. For example for station number k:

$$\text{if } (x_i, y_j) \notin AF_k \Rightarrow Cij \neq f(W_k) \qquad (4.3)$$

The most commonly used weighting functions are as follows:

(i) $W(r) = \frac{R^2 - r^2}{R^2 + r^2}$ (Cressman, 1959)

(ii) $W(r) = \exp(-0.1r^2)$ (MacCracken and Sauter, 1975) - a Gaussian weighting scheme, which eliminates the dominance of a measuring station near a grid point (used by Dickerson, 1978).

(iii) $W(r) = \frac{1}{r^a}$ (used by Sherman, 1978; Goodin et al., 1980).

Different values for a may be chosen. The most common is to use distance-squared weighting $W(r) = \frac{1}{r^2}$. Figure 2 shows some results of interpolation (for a one-dimensional case) using $a = 1,2,10$. Figure 3 presents some functional forms of $W(r) = f(\frac{r}{R})$ which are used in practice.

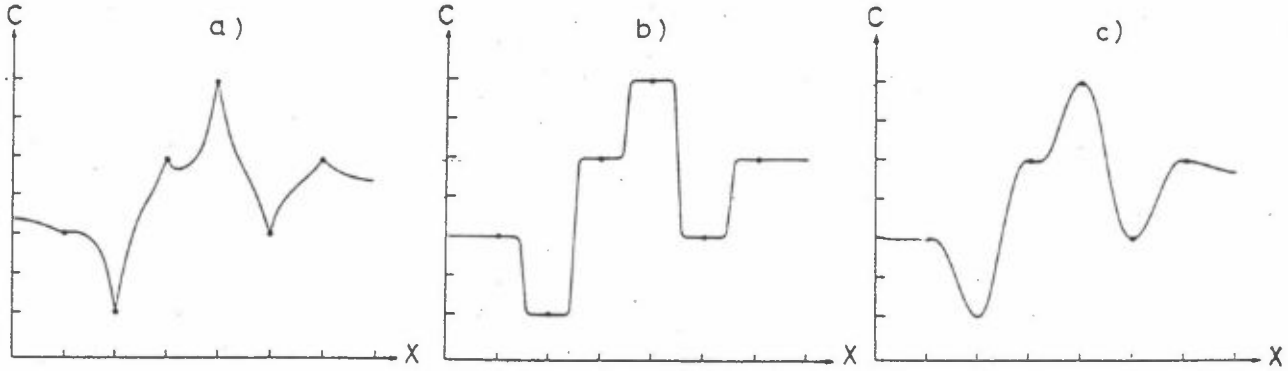

Figure 2: Results of using different exponents in the weighting function $W(r) = 1/r^a$. The three cases correspond to (a) 1/r. (b) $1/r^{10}$ and (c) $1/r^2$ (after Gordon and Wixom, 1978).

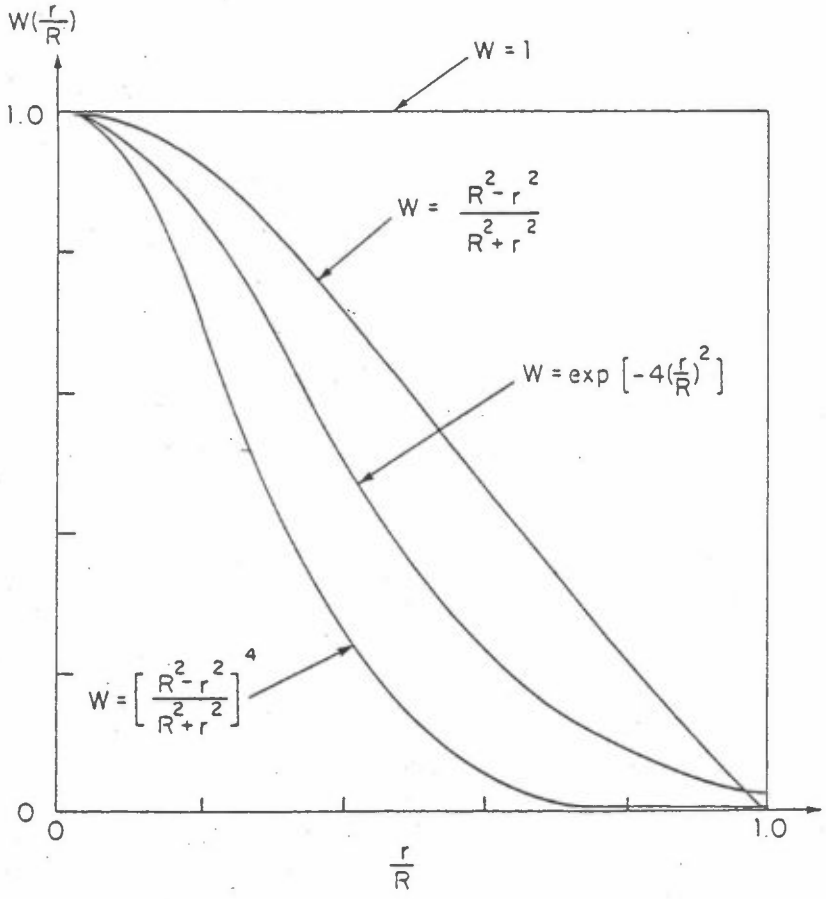

Figure 3: Some examples of different weighting functions of the form W(r/R), where R is the radius of influence (after McRae et al, 1982).

In most practical applications, the choice of an optimum radius of influence (R), must be based on the average station separation (s). For a 2-D area A, with n stations randomly distributed, we have:

$$d = \frac{n}{A} \quad \text{- average station density} \tag{4.4}$$

$$s = \left(\frac{A}{n}\right)^{\frac{1}{2}} \quad \text{- average station separation} \tag{4.5}$$

It has been found that:

- the optimum search radius R for large signal-to-noise ratios is

  $\frac{R}{s} \cong 1.6$ (Stephens and Stitt, 1970) (4.6)

- the value of R should be overestimated rather than underestimated

- each grid point must be within the radius of influence of at least one measurement station

b. Least-squares polynomial interpolation

The technique is based on the fitting of a polynomial by the method of least squares to the data points in an area surrounding the grid point at which the value is required.

A second-degree polynomial, for example, can be expressed in the form:

$$C'_k = a_{00} + a_{10}x_k + a_{01}y_k + a_{11}x_k y_k + a_{20}x_k^2 + a_{02}y_k^2 \tag{4.7}$$

where the (x_k, y_k) are the measurement points ($k=1,2,\ldots,n$; $n \geq 6$) and C'_k are the estimated values of C_k values at these points. In order to determine the six coefficients a_{ij}, independent observations of C_k at, at least 6 points, are clearly required.

The a_{ij} coefficients have to be chosen so that the following form is minimized:

$$S = \sum_{k=1}^{n} (C_k - C_k')^2 = \sum_{k=1}^{n} [C_k - \sum_{i+j=0}^{i+j=2} a_{ij} x_k^i y_k^j]^2 \qquad (4.8)$$

where C_k are the measured values for $k = 1,2,...,n$ ($n \geq 6$).

Thus we have to solve the set of six linear algebraic equations

$$\frac{\partial S}{\partial a_{ij}} = 0; \text{ where } 0 \leq i+j \leq 2 \qquad (4.9)$$

for the coefficients a_{ij}. Having calculated the a_{ij} values, the wind speed at any grid location (x,y) can be computed from:

$$C(x,y) = \sum_{i+j=0}^{i+j=2} a_{ij} x^i y^j \qquad (4.10)$$

Higher-order polynomials can be used, but it requires more coefficients (that means more available data) and much more computer time to solve the coefficient matrix. (The computer time is approximately proportional to m^2, when m is the number of matrix elements).

Instead of polynomial interpolation for the entire grid, subregions of the grid can be used. A simpler technique is to triangulate the region, using the station locations for the vertex positions. Another possibility is to use the planar divisions called Voroni or Thiessen polygons, which are based on the assumption that each station measurement is associated with the local region of the area nearer to the station in question, than to any other.

c. Optimum interpolation

This general technique was first presented by Gandin (1965). The interpolation function is determined by the statistical properties of the wind field (covariance, correlation function). In order to calculate these statistical properties, historical records of data values have to be available.
This method was used by Eidsvik (1978, 1981) to estimate the wind-field over Oslo.

d. Summary

A short summary of the methods mentioned, is given in Table 1.

Table 1: Summary of interpolation methods and their attributes. (After Goodin et al., 1979).

Method	Attributes	Applications
Weighted interpolation (a) $W(r) = (\frac{R^2-r^2}{R^2+r^2})^m$ (b) $W(r) = \exp(-ar^m)$ (c) $W(r) = r^{-m}$	1. Easy to implement 2. May be modified if directional influence is important 3. More features of the data are smoothed as m decreases 4. Influence of a station becomes very localized as m (or a) increases 5. Radius of influence R may be fixed or variable	Cressman (1959) Endlich & Mancuso (1968) Shepard (1968) MacCracken & Sauter (1975) Hovland et al. (1977) Boone & Samuelson (1977)
Least-squares polynomial interpolation (a) Polynomial of degree m fitted to full grid (b) Polynomials of degree m fitted to subregions of grid	1. Complex to implement 2. Resulting field depends strongly on distribution of data points when using (a) 3. Resulting field is smoothed as m decreases, when using (a) 4. (b) fits data almost exactly 5. Execution time increases with m	Panofsky (1949) Akima (1975) McLain (1974,1976) Lawson (1977)
Optimum interpolation	1. Complex to implement 2. Much historical data may be required 3. Statistics of the data assounted for	Gandin (1963) Dartt (1972) Schlatter (1975)

Franke (1977) has carried out an extensive comparison between eleven interpolation functions. His conclusion is that six points per region appears to give sufficient function definition. He recommended one of McLain's (1974) distance weighted least squares methods. In this method the fitting function is a second degree

polynomial, and a weight (in form $W(r) = \frac{\exp(r)}{r}$) is attached to each data point (x_k, y_k).

Goodin et al. (1979) have made a comparison of the interpolation methods applied to wind fields. They found that a second-degree polynomial fitting procedure with r^{-2} distance-weighting scheme, provided the best results.

e. Influence of topography

In the interpolation procedure, the influence of topography can be included to some extent. Shepard (1968) defined an effective distance r' by:

$$r' = [r^2 + b(r)^2]^{\frac{1}{2}} \qquad (4.11)$$

where:

r - distance between measurement points (x_k, y_k) and grid point (x_i, y_j)

b(r) - length of "barrier" perpendicular to the line between (x_i, y_j) and (x_k, y_k)

This concept is shown in Figure 4.

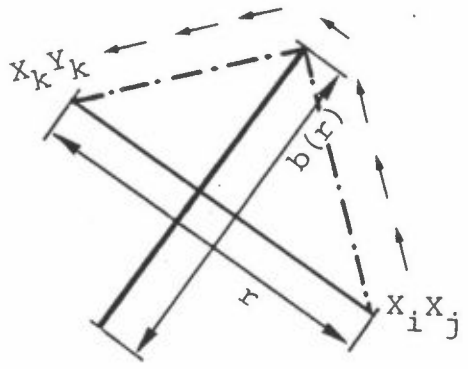

Figure 4: Concept of computing an effective distance between points which are separated by "barrier".

This procedure can be used to include the influence of gross terrain features that have dimensions more than one grid cell length.

4.3 Objective analysis procedure

Once the initial wind flow has been established, the kinetic properties of the field are adjusted in the second step. As mentioned before, the constraint which has to be satisfied by the initial wind field is the continuity equation in the Boussinesq form:

$$\text{div } \bar{U}_o = 0$$

In other words, the divergence of the initial wind field has to be reduced.

In this second step of the analysis, the values of the wind obtained from the first step at each grid point, are assumed to represent observed values at these points ($\bar{U}_o$ - observed wind vector).

Endlich (1967) used a point-iterative method (for 2D field) in which mesh values are altered locally to reduce two-dimensional divergence and retain the original vorticity of the field. This method has several advantages - explicit calculation of boundary conditions is not required, computations are relatively simple (10-15 iteration is needed to remove divergence) and converge well. Recently, van Egmond and Kesseboom (1983) used this method for smoothing the wind field in their mesoscale Eulerian grid model for the Netherlands.

Liu and Goodin (1976) have proposed an iterative algorithm, similar to that of Endlich, in which the wind vectors at the measuring stations are held fixed as additional constraints.

Sherman (1978) - for a three dimensional wind field called MATHEW;
Dickerson (1978) - for a 2D vertically integrated version of

MATHEW called MASCON; McEwen et al. (1980) - for a 2D model; Davis and Bunker (1980), Pudykiewicz (1981) and Racher et al. (1983) - for 3D models, have all adapted the variational calculus approach of Sasaki (1958,1970) for generating a mass-consistant flow.

In the Sasaki method, the general variational analysis formalism defines an integral function whose extremal solution minimizes the variance of the difference between the initial and updated values of a variable, subject to physical constraints, in a least square manner. In this scheme subsidiary conditions which are to be satisfied exactly, are known as strong constraints; while conditions that are only approximately imposed, are weak constraints.

In order to obtain a mass-consistant flow, the continuity equation is used as a strong constraint

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (4.12)$$

The variational functional for a 3D approach may be expressed as follows (Sherman, 1978):

$$E(u,v,w,\lambda)=\int_{x,y,z} [\alpha_1^2(u-u_o)^2+\alpha_1^2(v-v_o)^2+\alpha_2^2(w-w_o)^2+\lambda(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z})]dxdydz \qquad (4.13)$$

where:

λ = the Lagrangian multiplier, which is a function of observational errors and deviations from the constraint,

α^2= the Gaussian precision moduli defined by $\alpha^2=\frac{1}{2}\sigma^{-2}$, where σ^2 is the error variance of the observed field, and α_1^2 and α_2^2 are related to the horizontal and vertical wind velocities, respectively.

$U(u_o,v_ow_o)$ - observed wind vector (w_o is usually assumed to be zero)

$U(u,v,w)$ - adjusted wind vector

The Euler-Lagrange equations associated with the minimalization of (4.13) are:

$$u = u_o + \frac{1}{2\alpha_1^2} \frac{\partial \lambda}{\partial x} \tag{4.14}$$

$$v = v_o + \frac{1}{2\alpha_1^2} \frac{\partial \lambda}{\partial y} \tag{4.15}$$

$$w = w_o + \frac{1}{2\alpha_2^2} \frac{\partial \lambda}{\partial z} \tag{4.16}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4.17}$$

Substitution of (4.14-4.16) into (4.17) yields an equation for λ:

$$\frac{\partial^2 \lambda}{\partial x^2} + \frac{\partial^2 \lambda}{\partial y^2} + \left(\frac{\alpha_1^2}{\alpha_2^2}\right) \frac{\partial^2 \lambda}{\partial z^2} = -2\alpha_1^2 \left(\frac{\partial u_o}{\partial x} + \frac{\partial v_o}{\partial y} + \frac{\partial w_o}{\partial z}\right) \tag{4.18}$$

Equation (4.18) is solved for λ with boundary conditions, and the adjusted velocity field is calculated from (4.14-4.16).

At the boundary either λ or the variation of the normal velocity component $(\frac{\partial \lambda}{\partial n})$ must be zero.
The condition $\lambda = 0$ is used for open or "flow-through" boundaries, while $\frac{\partial \lambda}{\partial n} = 0$ describes closed or "no-flow-through" boundaries.

The adjustment process is sensitive to the value chosen for $(\alpha_1/\alpha_2)^2 \equiv (\sigma_2/\sigma_1)^2$. This value should be proportional to the magnitude of the expected $(w/u)^2$ (Sherman, 1978). For neutral conditions, $(\alpha_1/\alpha_2)^2 \cong 10^{-4}$. If this ratio is larger, the adjustment is predominantly in the vertical component; if it is smaller, the horizontal adjustment dominates.

More recently, Goodin et al. (1980) proposed a new procedure, which avoids most of the limitations of the algorithms presented before. A comparison between this method and the MATHEW wind field of Sherman (1978), is given in table 2.

Table 2: Comparison of attributes of three-dimensional divergence reduction procedures. (After Goodin et al., 1980).

Attribute	New technique (Goodin et al., 1980)	MATHEW (Sherman, 1978)
Coordinate system	Terrain-following coordinates	Coordinate system parallel to sea level
Treatment of flow over complex terrain	Barriers to flow are used during procedure. Surface layer flow is adjusted using $\nabla^2\phi=D$, where D is magnitude of vertical perturbation (see section 6)	Obstacle cells are used to represent terrain. They are treated as no-flow-through boundaries
Interpolation procedure	$1/r^2$-weighting of station data at surface. 1/r-weighting at each level above surface	$1/r^2$-weighting at surface. Upper level values are obtained from synoptic analysis
Treatment of horizontal boundary conditions during divergence reduction procedure	Normal component of velocity at boundary is adjusted according to value at adjacent interior point. (Same procedure as at all other interior points)	Program accepts $\partial\lambda/\partial n = 0$ or $\lambda = 0$ as boundary conditions. Derivative is approximated by three-point difference.
Treatment of atmospheric stability	Number of smoothing passes through the interpolated field at each level is related to the stability at that level. Amount of smoothing required for a given stability class is obtained empirically	Gaussian precision moduli, α_1,α_2, which are functions of measurement errors, must be determined empirically.
Variable vertical grid	Yes	No
Computer time required	25 000 points (100x50x5) Divergence $\rightarrow 10^{-4}s^{-1}$ 5 min on IBM 370	23 000 points (25x33x28) Divergence $\rightarrow 10^{-12}s^{-1}$ 2-5 min on CDC 7600 (20-50 min on IBM 370)

The divergence reduction procedure proposed by Goodin et al., (1980) involves three steps:

1) To reduce as much of the anomalous divergence as possible, at each level above the lowest layer, a simple five point smoothing filter, is used (both for the u and v field):

$$u_{ij}^{n+1} = 0.20(u_{ij}^{n}+u_{i+1,j}^{n}+u_{i-1,j}^{n}+u_{i,j+1}^{n}+u_{i,j-1}^{n})(1-\beta_k)+\beta_k u_{i,j}^{n} \qquad (4.19)$$

where β_k is a parameter which allows the user to keep the measured velocity at station k fixed (β_k=1) or keep only some of its original influence (β_k<1). β_k=0 at all non-measuring station points.
The number of passes through the smoothing step is related to the atmospheric stability, and must be determined empirically.

2) The vertical velocity above each level is computed from the divergence within that layer, and held fixed throughout the rest of the procedure.

3) The remaining divergence which exists within each layer is reduced by application of a two-dimensional technique similar to that of Liu and Goodin (1976).

It was found that 100 iterations were sufficient to refine the divergence reduction during the second step.

The above techniques require an empirically determined parameter: $(\alpha_1/\alpha_2)^2$ in the approach of Sasaki and number of passes in the method of Goodin et al. (1980). A major advantage of the last technique is that it allows the boundary values to adjust in response to the interior flow.

5 SHORT ALGORITHM FOR CONSTRUCTING WIND FIELDS

5.1

The procedure for generating a two dimensional (2D) wind field involves the following steps:

A. The spatially sparse surface measurements are adjusted to a constant height above the surface using a standard power law

$$u_2 = u_1\left(\frac{z_2}{z_1}\right)^p \qquad (5.1)$$

the exponent p is determined by the atmospheric stability condtions, u_1 is the measured wind velocity at a reference height z_1.

B. One of the interpolation methods is used to obtain an initial surface wind field from the available surface wind measurements.

C. Local terrain-adjustment or thermal-adjustment techniques are employed to describe the "disturbance field", which is added to the initial surface wind field (see section 6).

D. One of the objective analysis procedures are used to adjust the initial "disturbed" wind field such that a mass-consistant surface wind field is produced.

5.2

The first part of procedure for generating a three-dimensional wind field consists of the A, B, C, (or A,B,C,D) steps from the 2D approach, and gives the initial surface mean wind field (or adjusted 2D wind field) as a starting point for the 3D calculations. The next steps are as follows:

E: An initial 3D wind field is estimated using one of the methods described in section 4.1.

F. This field is adjusted, while preserving mass-concervation, either by minimalization of the remaining divergence of the whole 3D field or by reducing the divergence at each horizontal level while maintaining a small vertical velocity above each level.

A terrain-following vertical coordinate (sigma-coordinate) and a variable vertical grid spacing can be applied to this scheme.

6 MESOSCALE INFLUENCES ON WIND FIELD

Anderson (1971) has proposed a 2D surface model which includes the perturbations of the wind field due to topographic and thermal effects. Each of these effects are treated separately and the solutions are added to obtain the net wind-field disturbance (see point 5.1; C). This adjustment procedure involves solution of the Poisson equation:

$$\nabla^2\phi = f(x,y) \qquad (6.1)$$

where:

ϕ - potential function of the perturbation
$f(x,y)$ - forcing function

In the terrain-adjustment case, the forcing function can be expressed as follows:

$$f(x,y) = \frac{1}{H} U\nabla h \qquad (6.2)$$

where:

H - upper boundary of the disturbed air
h - local altitude of the surface
U - unperturbed mean velocity
$\nabla = [\frac{\partial}{\partial x}, \frac{\partial}{\partial y}]$

For the thermal effect, we can write:

$$f(x,y) = \frac{A(T_g - \bar{T})}{H} \qquad (6.3)$$

where:

A - constant of proportionality to be determined from observations
T_g - ground temperature
$\bar{T}$ - spatial mean value of T_g

Common applications of "thermal fields" are in connection with "heat island" in towns and thermal effects of various terrain types ("land-sea" breeze, "lake breeze" effects.)

The net wind field is calculated from

$$\bar{\bar{V}} = \nabla\phi \tag{6.4}$$

where potential is composed of three components:

$$\phi = \phi_{mean} + \phi_{topographic} + \phi_{thermal} \tag{6.5}$$

In addition Anderson made the assumption, that the mean wind prevails outside of and on the boundary, but is perturbed within the region. This corresponds to zero potential at the boundary.

7 NUMERICAL SOLUTIONS

An important element of the wind field generation procedure involves repeated solutions of the Poisson equation and its associated boundary condition. Over the two-dimensional rectangle R(nxm), the system can be written as:

$$\nabla^2 f = g \quad (x,y) \in R$$

with the Dirichlet boundary condition (7.1)

$$f = 0 \quad (x,y) \in \partial R$$

where f and g are functions of x and y. The considered area of computation is shown in Figure 5.

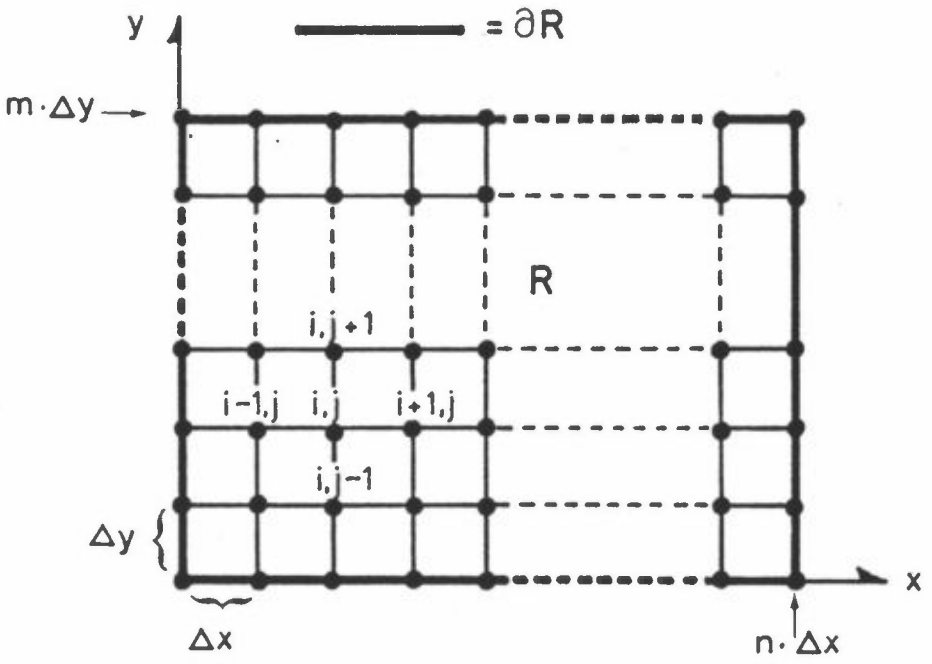

Figure 5: Area of computation with the grid system.

Because solution of (7.1) is repeated many times, it is important to choose the most efficient numerical method.
Methods for solving partial differential equations are not directly applicable to the system (7.1), but only to its finite difference approximation. The second-order finite difference approximation scheme of (7.1) can be expressed as (McRae et al., 1982):

$$\frac{f_{i-1,j} - 2f_{i,j} + f_{i+1,j}}{(\Delta x)^2} + \frac{f_{i,j-1} - 2f_{i,j} + f_{i,j+1}}{(\Delta y)^2} = g_{ij} \qquad \begin{matrix} 2 < i < n-1 \\ 2 < j < m-1 \end{matrix} \qquad (7.2)$$

$$f_{o,j} = 0 \qquad f_{n,j} = 0 \qquad 1 < j < m$$
$$f_{i,o} = 0 \qquad f_{i,m} = 0 \qquad 1 < i < n \qquad (7.3)$$

If $\Delta x = \Delta y$, a block tridiagonal form can be used:

$$[M]\ \underline{f} = \underline{h} \qquad (7.4)$$

where the matrix M is given by:

$$M = \begin{bmatrix} D & I & & & \\ I & D & . & & \\ & . & . & . & \\ & & . & . & I \\ & & & I & D \end{bmatrix}_{(n-2)x(m-2)} \qquad (7.5)$$

and is composed of unit matrices I(n-2) and matrices D of the form:

$$D = \begin{bmatrix} -4 & 1 & & & \\ 1 & -4 & . & & \\ & . & . & . & \\ & & . & . & . \\ & & & . & . & 1 \\ & & & & 1 & -4 \end{bmatrix} \qquad (7.6)$$

The numerical methods of solving the finite-difference equation (7.2) fall into two groups:

A. Direct methods (exact solutions)

Such methods were at first applied with the restriction that the number of grid points, (n or m), could be expressed as $n = 2^k$ ($k = 1,2,3..$). More recently, direct methods have been extended to other, even irregular regions (Temperton, 1979). The direct methods include:

- methods based on Fourier decomposition in one dimension, using fast Fourier transform (FFT) techniques.

- methods based on block-cyclic reduction

In addition to these two methods, FACR(ℓ) algorithm should be presented, which combines first two methods.

- FACR(ℓ) - Fourier analysis-cyclic reduction algorithm, which begins with ℓ steps of cyclic reduction and resulting equations are then solved via the method of Fourier analysis.

The remaining unknowns are obtained by ℓ steps of the back substitution phase.

Advantages: solution does not involve any iterative steps and requires minimal storage

FACR(ℓ) is extremely fast. Swarztrauber (1977) has compared that for a 63 x 63 grid, the solution via the FACR algorithm requires roughly the same amount of time as five SOR iterations (see next section).

(Used by McRae et al., 1982; Pudykiewicz, 1981).

B. Iterative procedures (approximate solutions)

An initial guess of f^{o}_{ij} is needed, and this value is then corrected (according to a prescribed rule) during an iterative procedure until suitable convergence has been achieved. The most important methods are:

- SOR - successive over relaxation method, which is a point iterative technique, involving successively applied local corrections to improve an approximate solution. The k+1 iteration can be written (for $\Delta x = \Delta y$):

$$f^{k+1}_{ij}=(1-\omega)f^{k}_{ij}+\frac{\omega}{4}[f^{k}_{i+1,j}+f^{k}_{i-1,j}+f^{k}_{i,j+1}+f^{k}_{i,j-1}-\Delta x^{2}g_{i,j}] \qquad (7.7)$$

where ω is a relaxation factor used to accelerate convergence of the iteration process ($1 \leq \omega \leq 2$). It was found that optimum value for ω is $1.6<\omega<1.8$.

A modification of this method in which the value of ω is changed at each half-iteration, is called cyclic Chebyshev semi-iterative method (CCSI).

Advantages: easy to program, has minimal storage requirements

(Used by Sherman, 1978 in MATHEW and by McEwen et al., 1980).

- Gauss-Seidel method, which is a SOR method with $\omega = 1$.

Advantages: easy to implement.

(Used by Endlich, 1967).

- ADI - alternating direction implicit method, which is a two-step technique involving the solution of a tridiagonal set of equations along lines parallel to the x- and y-axes at the first and second step respectively.

Advantages: requires little storage, very fast rates of convergence may be attained.

(Chosen by Goodin et al., 1980, as the most efficient in comparison to SOR and Fourier series method).

C. Comparison

Since 1965 it has become increasingly popular to solve discretized Poisson equations by direct rather than iterative methods. Direct techniques yield the exact solutions and do not accumulate errors, which may happen in time-dependent models when iterative methods are used (because of first-guess errors).

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Methods for constructing mass-consistant wind fields are presented. Both two- and three dimensional cases are considered. Initial wind field generation, adjustment procedures, mesoscale influences and numerical solutions are described.

**Kategorier: Åpen - kan bestilles fra NILU A
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