NILU
TEKNISK NOTAT NR. 11/80
REFERANSE: 0680
DATO: AUGUST 1980

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ISBN 82-7247-186-8

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THE APPLICATION OF GAUSSIAN DISPERSION MODELS AT NILU

1. INTRODUCTION

This report presents the basics for the use of Gaussian type dispersion models in use at the Norwegian Institute for Air Research (NILU). It is taken for granted that the theoretical background for these models which has been published in a large number of text book and papers (1, 2, 4, 6), is well known.

2. THE GAUSSIAN PLUME EQUATION

The so-called Gaussian plume equation describes mathematically how to calculate the concentration "c" of a gas or a gas-equivalent air pollutant (e.g. dust with particle sizes of less than 10 μm) being emitted from a single source continuously:

$$c(x,y,z) = \frac{Q}{2\pi\bar{u}\cdot\sigma_y(x)\cdot\sigma_z(x)} \exp\left(-\frac{y^2}{2\sigma_y^2(x)}\right). \qquad (1)$$

$$\left\{\exp\left(-\frac{(z-h)^2}{2\sigma_z^2(x)}\right) + \exp\left(-\frac{(z+h)^2}{2\sigma_z^2(x)}\right)\right\}$$

where:

(x,y,z) = location of receptor point given in rectangular coordinates with the origin at ground level at the source location and x-axis parallel to wind direction

Q = continuous source emission rate at the air pollutant

h = effective plume height (stack height (h_s)+plume rise (dh).

$\bar{u}$ = mean transport (wind) speed

$\left.\begin{matrix}\sigma_y(x)\\ \sigma_z(x)\end{matrix}\right\}$ = diffusion parameters

This equation is an analytical solution of the simplified diffusion equation

$$\bar{u} \frac{\partial c}{\partial x} = k_z \frac{\partial^2 c}{\partial z^2} + k_y \frac{\partial^2 c}{\partial y^2} , \qquad (2)$$

assuming Gaussian distributions of the pollutant concentration in the plume normal to the drift direction of the plume.

It is furthermore assumed that:

- the pollutant transfer by advection in the transport direction is greater than by turbulent diffusion;
- steady state conditions are prevailing, which implies that all variables and parameters are constant in time;
- k_y and k_z are constant in the x, y and z directions;
- no uptake or deposition at the ground occurs; this means, that the plume can be described mathematically as completely reflected at the ground level which is assumed to be flat.

The assumption which neglects the turbulent diffusion in the drift direction relative to advection implies that the Gaussian plume equation should usually be applied for average transport speeds of more than 1 m/s.

The use of a constant average transport speed and a fixed wind direction during the basic time period reflects the assumption of a stationary and homogeneous horizontal wind field. Directional wind shear in the boundary layer is not considered.

The Gaussian plume equation, therefore, can only calculate short term concentrations over basic time periods (of about ½ to 1 hour duration) for which there are no significant changes of wind direction and speed, and which can be represented by the diffusion parameters σ_y and σ_z.

The basic model does not consider the plume history, i.e. each basic time period is completely independent.

If calculations are to be performed at ground level ($z = 0$) only, equation *(1)* reduces to

$$c(x,y,z = 0) = \frac{Q}{\pi \cdot \bar{u} \cdot \sigma_y(x) \cdot \sigma_z(x)} \exp\left(- \frac{y^2}{2\sigma_y^2(x)}\right) . \exp\left(- \frac{h^2}{2\sigma_z^2(x)}\right) \qquad (3)$$

Vertical diffusion of a plume by turbulent mixing is limited both by the earth's surface and often by the existence of a stable layer of air aloft, i.e. an inversion layer (mixing height).

3. ELEMENTS OF THE GAUSSIAN PLUME EQUATION

3.1 Wind Speed

The mean transport velocity should be representative of the conditions throughout the vertical height interval in which the plume is dispersing. The wind speed in the lower atmosphere varies with height above ground, however. Since wind measurements are generally performed near ground level only (10 meters), an adjustment for the expected height range of dispersion has to be made. The variation of wind speed with height depends also on the atmospheric stability.

The height dependence of the wind speed is described by a power law:

$$\bar{u}\ (z) = \bar{u}\ (z_o)\ (\frac{z}{z_o})^m \qquad (4)$$

with

z = height above ground,
z_o = reference height above ground,
$\bar{u}$ = time average wind speed,
m = wind profile exponent:

$$\bar{u}(z < z_o) = u(z_o)$$

The mean transport speed representative of an appropriate height range, e.g. from the effective source height (h) to ground level (for dispersion calculations), or from physical (hs) to the effective source height (for plume rise calculations), may then be calculated via integration:

$$\bar{u} = \frac{1}{\Delta z} \int \bar{u}\ (z)\ dz = \frac{1}{(z_2-z_1)} \int_{z_1}^{z_2} \bar{u}\ (z_o) \cdot (\frac{z}{z_o})^m\ dz, \qquad (5)$$

where

z_1 = 0 or h_s,
z_2 = h

Several empirical values of wind profile exponents (m) for different turbulence conditions have been published (1,2,6). In the NILU models the following values have been applied:

	m
Unstable	0.20
Neutral	0.28
Slightly stable	0.36
Stable	0.42

3.2 Atmospheric stability

The diffusion of air pollutants in the lower atmosphere is strongly influenced by the local atmospheric stability. The unstable atmosphere disperses effluent more rapidly than the stable atmosphere.

The stability of the atmosphere can be derived from vertical and horizontal turbulence measurements, or from measurements of the vertical temperature profile and wind speed. Estimates of the net radiation or cloud cover, ceiling height, and solar elevation have also been used. For practical reasons the turbulence situations of the atmosphere is usually described by a discrete set of stability classes.

Different turbulent classification schemes have been developed and used (1,3,4,5,6). Pasquill (7) defined 6 turbulence classes:

A = extremely unstable
B = moderately unstable
C = slightly unstable
D = neutral
E = slightly stable
F = moderately stable

The meteorological data used to determine the turbulence type are usually the surface wind speed, daytime insolation, and nighttime cloudiness.

In the NILU data input for dispersion models, the 3 unstable classes have been combined into one. The stability classes are usually defined by vertical temperature gradients and by direct measurements of the standard deviation of the horizontal wind direction fluctuations, where such data are available. The stability classes are defined as follows:

Class	Temperature gradient dT (deg/100 m)	Corresponds to:		
		Pasquill (5)	Klug (4)	Brookhaven (6)
Unstable	$dT < 1$	A + B + C	IV+V	$B_1 + B_2$
Neutral	$-1 \leq dT < 0$	D	$III_1 + III_2$	C
Slightly stab.	$0 \leq dT < 1$	E	II	-
Stable	$dT \geq 1$	F	I	D

3.3 Diffusion Parameters

A main assumption for solving the diffusion equation is the existence of a Gaussian normal distribution of the plume concentrations perpendicular to the transport direction. The diffusion parameters σ_y and σ_z are defined as the standard deviations of these Gaussian distributions. They are functions of the downwind distance from the emission source and of the stability of the atmosphere. The standard deviations have been determined from tracer experiments (8,9,10,11) or measurements of the wind fluctuation (12,13,14).

The most appropriate set of diffusion parameters should be selected for a particular application. The choice will be dependent upon source height, surface roughness and, in some cases, averaging time or transport distance. A set of different parameters has been evaluated at NILU (15) and represents the basis for our selection of parameters.

When wind fluctuation measurements are not available, the following form of diffusion parameters is used:

$$\sigma_y(x) = ax^p, \qquad \sigma_z(x) = bx^q. \tag{6}$$

The most commonly used coefficients are listed below, and apply to averaging times of up to one hour.

Source and surface specifications	Coefficients	Unst.	Neutr.	Sl. stable	Stable	Ref.
Surface	a	0.31	0.22	0.24	0.27	(1)
Emission	p	0.89	0.80	0.69	0.59	(5)
Low stacks	b	0.07	0.10	0.22	0.26	
Smooth surface	q	1.02	0.80	0.61	0.50	
Surface and low	a	1.7	0.91	1.02	-	(10)
sources (area sources)	p	0.72	0.73	0.65	-	
Rough surface,	b	0.08	0.91	1.93	-	
urban	q	1.2	0.70	0.47	-	
High stacks	a	0.36	0.32	0.31	0.31	(6)
Smooth to	p	0.86	0.78	0.74	0.71	
medium rough	b	0.33	0.22	0.16	0.06	
surface	q	0.86	0.78	0.74	0.71	
High stacks	a	0.23	0.22	1.69	5.38	(11)
Rough surface	p	0.97	0.91	0.62	0.57	
	b	0.16	0.40	0.16	0.40	
	q	1.02	0.76	0.81	0.62	

For cases where the standard deviations of wind fluctuations are available, the following expression is used (12):

$$\sigma_y = \sigma_\theta \cdot x \cdot f\ (t/t_L), \qquad (7)$$

where:

$$f = (1+0.055t^{0.5})^{-1} \text{ for a roughness lengt of} \sim 5\text{-}10 \text{ cm} \qquad (8)$$

$$f = 4.6 \cdot t^{-0.33} \text{ for roughness lengt of} \sim 0.5 \text{ m} \qquad (9)$$

σ_θ = the standard deviation of horizontal wind fluctuations (rad),
t = the transport time (x/u).

3.4 Effective Source Height

The concentration of air pollutants in the vicinity of a source depends strongly on the release height and the plume rise. The sum of both is often called the "effective source height".
The plume rise is influenced by the difference in temperature between the exhaust gas and the ambient air, the wind speed, the exit gas velocity, and the stability of the atmosphere. Many attempts have been made to describe mathematically the plume behaviour immediately after it has left the source, However, there is not yet an ideal general formula available.

Based upon a consideration of several plume rise equations (16), a set of formulas for the plume rise (dh) has been selected for the NILU type Gaussian dispersion models.

For small sources (heat output, $Q_h < 2 \cdot 10^5$ cal/s),

Holland (17): $dh = (1.5 \cdot d \cdot w + 4 \cdot 10^{-5}\ Q_h) \cdot u^{-1}$ (10)

For medium sized sources and industrial sources,

$(2 \cdot 10^5 < Q_h < 7 \cdot 10^6$ cal/s).

Stümke (18) : $dh = (1.5 \cdot d \cdot w + 65 \cdot d^{3/2}((T_s - T)/T_s)^{0.25}) \cdot u^{-1}$ (11)

Bringfelt (19): $$\left.\begin{aligned} dh &= 167 \cdot Q_{MW}^{0.36} \cdot u_s^{-1} \quad (\text{for } x = 500 \text{ m}) \\ dh &= 224 \cdot Q_{MW}^{0.34} \cdot u_s^{-1} \quad (\text{for } x = 1000 \text{ m}) \end{aligned}\right\} \quad (12)$$

where Q_{MW} = heat output in MW ($\simeq 0.11 \cdot F$)

For large sources, tall stacks, power plants ($Q_h > 7 \cdot 10^6$ cal/s),

$$\left.\begin{aligned} &dh = 1.6 \cdot F^{1/3} \cdot x^{2/3} \cdot u^{-1} \\ &\qquad \text{for } x \leq 10\ h_s \\ &dh = 1.6 \cdot F^{1/3} \cdot (10\ h_s)^{2/3} \cdot u^{-1} \\ &\qquad \text{for } x > 10\ h_s \end{aligned}\right\} \text{for unstable and neutral cases.} \quad (13)$$

$$dh = 2.9 \cdot (F/(u \cdot s))^{0.33}, \text{ for stable cases} \quad (14)$$

$$F = g \cdot w(d/2)^2 (T_s - T)/T_s, \quad s = (g/T)\ (\partial\theta/\partial z) \quad (15)$$

3.5 Physical or Chemical Transformations

When solving the simplified diffusion equation it is assumed that the mass of air pollutants is conserved throughout the transport process. However, air pollutants may undergo physical or chemical transformations. Such chemical transformations may result in the loss of a pollutant due to decay to another substance. Physical transformations, for example dry or wet deposition, adsorption or absorption may also be significant.

Dry deposition is taken care of by assuming a deposition velocity, V_d, which gives the amount of deposited material (D) proportional to the ground level concentration (C_o) (23):

$$D = V_d \cdot C_o \quad (16)$$

The deposition model is a so-called source depletion model, which reduces the source by an amount equal to that taken out by deposition. The source reduction factor for each distance increment, dx, is given by:

$$\frac{dQ^1_x}{dx} = - \int_{-\infty}^{\infty} D \, dy, \qquad (17)$$

which gives the reduced "source strength" at distance x from the source:

$$Q^1_x = Q[\exp \int_0^x (\sigma_z \cdot \exp(h^2/2\sigma_z^2))^{-1} \cdot dx]^{-(V_d \cdot u^{-1} \cdot \sqrt{(2/\pi)})} \qquad (18)$$

Wet deposition

Wet deposition might be accounted for by a first order decay process:

$$\frac{dc}{dt} = - \Lambda \cdot c$$

$$c' = c \cdot \exp(-\Lambda t) \qquad (19)$$

Λ is the washout coefficient

The washout rate is:

$$\omega = - \int_0^{\infty} (dc/dt) dz \simeq \frac{\Lambda \cdot Q_o \exp(-\Lambda t)}{\sqrt{2\pi} \cdot u \cdot \sigma_y} \exp(-y^2/2\sigma_y^2) \qquad (20)$$

Another subroutine assumes that the SO_2 in droplets is limited by the transformation of SO_2 to $H_2SO_3^-$ through

$$SO_2 + H_2O \rightleftarrows H_3O^+ + H_2SO_3^-$$

This means that the pH value of the raindrops, before reaching the plume, is essential for the SO_2-uptake rate.

The wash out is estimated from the centre plume concentration and from an assumption of raindrop pH-values. The model is based upon studies at Battelle Memorial institute in USA (24). This simple precipitation model includes approximations and uncertainties, and has thus been adjusted at large distances to match empirical wash out data from Sweden (25).

4 SPECIAL PROBLEMS

The Gaussian plume formula is applicable only for flat homogeneous terrain, steady state meteorological conditions and homogeneous turbulence.

To take into account other effects implies modifications and approximations. Nevertheless a few special cases might be approached by the Gaussian type models.

4.1 Topography

The effect of elevated terrain on the ground level concentrations might to a certain degree be reflected by reducing the effective plume hight (h) assuming:

$$h = h_s + dh \div k \cdot h_t \tag{21}$$

where h_t is the height of terrain above stack base level and k is a terrain factor $(0 < k < 1)$ dependent upon steepness, distance from source, stability etc.

4.2 Sea breeze fumigation

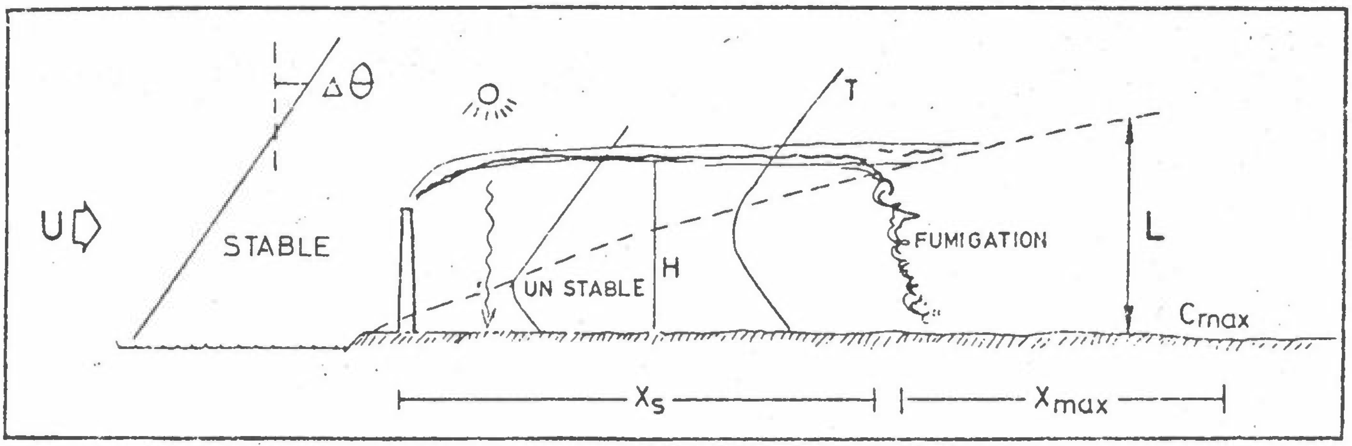

Figure 1: Sea breeze fumigation.

A rapid mixing downwards to the ground of pollutants in an elevated plume (fumigation), occurs when the stable sea breeze is advected inland and heated from below. The fumigation starts at the point where the unstable surface layer reaches the plume (see Figure 1).

The following method for estimating maximum ground level concentrations during these cases has been applied at NILU.

The height to the unstable layer (L) is given by van der Hovens (26):

$$L = 8.8 \cdot \sqrt{\frac{x}{u \cdot \Delta\theta}}, \qquad (22)$$

where:

x = distance from sea (m),
u = wind speed (m/s),
$\Delta\theta$ = vertical temperature gradient (deg/100 m).

Fumigation of the plume take place when $x = X_S$:

$$X_S = \left(\frac{H}{8.8}\right)^2 \cdot u \cdot \Delta\theta \qquad (23)$$

The increase of σ_z during the period of fumigation should be changed according to unstable dispersion (see Figure 1).

X_L is defined as the distance where the plume touches the ground. According to Turner (2):

$$2.15\ \sigma_z\ (X_L) = L. \quad \text{(As a first estimate: } L = H\text{)}$$

For unstable dispersion:

$$L = 8.8 \sqrt{\frac{X_L + X_S}{u \cdot \Delta\theta}}\ , \qquad (24)$$

$$X_L = (0.47 \cdot L/0.33)^{1.49} \qquad (25)$$

L and X_L are determined by iteration.

Maximum concentration occurs at a distance $x_{max} = 2X_L$ (2), which gives:

$$L(X_S + 2X_L) = 8.8 \sqrt{\frac{X_S + 2X_L}{u\Delta\theta}}\ , \qquad (26)$$

$$C_{max}\ \text{(sea breeze)} = \frac{Q}{\sqrt{2\Pi}\ \Sigma_y \cdot L(X_s + 2X_L) \cdot u}\ , \qquad (27)$$

$$\Sigma_y = \sigma_y\ \text{stab.}(X_s) + \sigma_y\ \text{unstab.}\ (X_{max}) \qquad (28)$$

5 MULTIPLE SOURCE HANDLING (NILU model "Kilder")

In general, calculations of air pollutant concentrations (at ground level) have to be made for a field of many sources. Under the restriction of no interdependence of the different sources a superposition of a number of sources emitting the same air pollutant is described by Schjoldager (21) and Sivertsen (22).

6 LIMITATIONS

Due to the assumptions of atmospheric homogenity and stationarity the Gaussian plume equation should only be applied over those distances for which the aforementioned assumptions could be expected to reasonably hold.

The Gaussian plume formula should also be used only for downwind distances for which the dispersion parameters have been determined experimentally (e.g. via tracer experiments) or semi-empirically (e.g. via wind fluctuation measurements). The use of dispersion parameters extrapolated to distances of more than some ten kilometers or of less than about 100 m can only show the tendency of the concentration values to be expected.

Calm wind situations (wind speed of about 1 m/s or less) cannot be handled by the Gaussian plume equation because at such low wind speeds the turbulent diffusion is equivalent to or even supersedes the advection influence.

The estimates of deposition or depletion at the ground cannot be taken into account without violation of the Gaussian hypothesis. Chemical reactions are not allowed in such a model. However, the exponential correction factor for a half-life time introduced, and the source depletion deposition estimates could be considered first order "guesstimates".

Dispersion is influenced by the physical structure of building complexes. For example, large objects can produce aerodynamic down wash, causing higher concentrations in their immediate vicinity. Additional assumptions then have to be made before using the Gaussian plume formula.

Uneven terrain like terrain steps, influences the air flow and therefore the strictly horizontal transport of pollutants, as assumed in the Gaussian plume equation, is unrealistic under such conditions. Corrections performed in the simple way given by equation (21) are uncertain, but reflects to a certain degree the effect of the topography.

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POSTBOKS 130, 2001 LILLESTRØM
ELVEGT. 52.
TLF. (02) 71 41 70

RAPPORTTYPE Teknisk notat	RAPPORTNR. TN 11/80	ISBN--82-7247-186-8
DATO August 1980	ANSV.SIGN. B. Ottar	ANT.SIDER 22
TITTEL The application of Gaussian dispersion models at NILU		PROSJEKTLEDER B.Sivertsen
		NILU PROSJEKT NR 0680
FORFATTER(E) Bjarne Sivertsen		TILGJENGELIGHET ** A
		OPPDRAGSGIVERS REF.
OPPDRAGSGIVER NILU		
3 STIKKORD (á maks.20 anslag) Spredningsmodeller | Gaussmodell		

REFERAT (maks. 300 anslag, 5-10 linjer)

Rapporten inneholder en oppsummering av parametre anvendt i Gaussiske spredningsmodeller ved NILU. Spredningsparametre, vindprofil, røykhevning, avsetning og spesielle meteorologiske effekter er beskrevet.

TITLE

ABSTRACT (max. 300 characters, 5-10 lines)

The report contains the basics for the use of Gaussian type dispersion models at NILU; dispersion parameters, wind profile, plume rise deposition and possible modifications.

**Kategorier:	Åpen - kan bestilles fra NILU	A
	Må bestilles gjennom oppdragsgiver	B
	Kan ikke utleveres	C