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A TIME DEPENDENT NUMERICAL DISPERSION MODEL FOR AIR POLLUTION, WITH APPLICATION TO THE CITY OF OSLO

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Summary: The vertically integrated continuity equation is solved numerically to calculate air pollution concentration over an urban area. Calculated and measured values are compared during an inversion situation in Oslo.

I INTRODUCTION

To study the air pollution level in urban areas, data describing the sources, the meteorological conditions, and the air pollution level, may be used to construct a diffusion model. R C Wanta in 1968 (1) and M Neiburger in 1970 (2) have given reviews of earlier works in this field. A Gaussian diffusion formula is often used to describe the pollution from single sources, and the total effect in an area is considered to be the sum of the effects from single sources.

As a solution of the mass continuity equation the Gaussian plume model is based on several simplifying assumptions. In many urban areas the major air pollution problems appear under meteorological conditions when these simplifying assumptions do not apply.To solve the continuity equation without simplifying assumptions, numerical methods have to be used.

A full understanding of the air pollution situations would require an understanding of the connection between local winds and local thermal effects, as related to topography. Usually pollution sources are also heat sources, therefore changes in the emission conditions may affect the wind field during the pollution situations. In order to study this effect, the energy and momentum equations should be incorporated in the numerical model.

The study reported here, uses observations of the horizontal wind to calculate the pollution level from the source distribution in Oslo, and only the continuity equation has been used.

In Oslo high air pollution levels appear during winter inversion situations characterized by a weak local wind field. In I969 H Reiquam considered the situation in Oslo as a transport problem and showed that model considerations might be useful. His way of calculation is published in Atmospheric Environment (3).

In an investigation of the air pollution in Oslo during the winter I969/I970 measurements during an inversion situation on the 26th and 27th of February I970 have been used to test a model based on a direct numerical solution of the continuity equation for SO_2. Actual data for SO_2-emission and wind are used.

The calculations are compared with measured SO_2-concentrations each half hour in the center of the city. The distribution throughout the city is compared with mean concentrations over 24 hours.

2 NUMERICAL SOLUTION OF THE CONTINUITY EQUATION

The continuity equation for dispersion of a pollution component in the atmosphere may be written:

$$\text{(I)} \quad \frac{\partial q}{\partial t} + \nabla\cdot(\vec{v}\, q) = \frac{\partial}{\partial x}\left(\rho K_x \frac{\partial q/\rho}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho K_y \frac{\partial q/\rho}{\partial y}\right) + \frac{\partial}{\partial z}\left(\rho K_z \frac{\partial q/\rho}{\partial z}\right) + \text{sources} + \text{sinks}$$

t	: time
x, y, z	: orthogonal coordinates with unity vectors $\vec{i}, \vec{j}, \vec{k}$.
q	: density of the pollution component
ρ	: density of the air
$K_x, K_y, K_z,$	: the turbulent diffusion coefficients along the x, y, z axes.
$v = u\vec{i}+u\vec{j}+w\vec{k}$	: the three-dimentional velocity
$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial j} + \vec{K}\frac{\partial}{\partial z}$	: the gradient operator

For the problem under consideration one has as a first order approximation put ρ = constant and $K_x = K_y = K$. As sources exist at many levels above the ground in an urban area, it is not practical to consider each single source. Instead the vertical mean of equation (1), from the ground to a level including the largest effective chimney height may be studied by integrating equation (1) with respect to z from the ground to a height H.

$$(2)\quad \frac{1}{H}\int_0^H \frac{\partial q}{\partial t}\,dz = -\frac{1}{H}\int_0^H \nabla_h \cdot (\vec{v}_h q)dz + \frac{1}{H}\int_0^H \nabla_h \cdot (K\nabla_h q)dz -$$

$$-\frac{1}{H}((wq)_H - (wq)_0) + \frac{1}{H}((K_z \frac{\partial q}{\partial z})_H - (K_z \frac{\partial q}{\partial z})_0) +$$

$$+\frac{1}{H}\int_0^H Q\,dz - \frac{1}{H}\int_0^H C_0\, q\,dz.$$

All sinks (i.e. deposition and oxidation from SO_2)to SO_3 are supposed to be proportional to concentration, and the factor of proportionality is denoted C_0. The other symbols are:

$Q\ (x,y,z,t)$	: the source strength
$\nabla_h = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y}$	: the horizontal gradient operator
$\vec{v}_h = u\vec{i} + v\vec{j}$	: the horizontal wind vector

At the ground $z = 0$, $w = 0$ and $K_z = 0$.

Introducing vertical mean quantities, denoted by ($\bar{}$), the following equation is obtained,

$$(3)\quad \frac{\partial(\bar{q})}{\partial t} = -\nabla_h \cdot \overline{(\vec{v}_h q)} + \nabla_h \cdot \overline{(K\ \nabla_h\ q)} - \frac{1}{H}\ (wq)_H +$$

$$+ (\frac{H_z}{H}\frac{\partial q}{\partial z})_H + \bar{Q} - C_0 q.$$

The vertical flux through the top of the air-volume under consideration is

$$(4)\quad F_H = \frac{1}{H}(wq)_H - (\frac{K_z}{H}\frac{\partial q}{\partial z})_H$$

The vertical flux is assumed to be zero at the ground. As a first approximation F_H is assumed to be proportional to the vertical mean concentration and combined with the sinks:

$$(5) \quad C\,\bar{q} = \overline{(C_0 q)} + \frac{1}{H}(wq)_H - \left(\frac{K_z}{H}\frac{\partial q}{\partial z}\right)_H$$

The equation may then be written,

$$(6) \quad \frac{\partial(\bar{q})}{\partial t} = -\nabla_h \cdot \overline{(\vec{v}_h q)} + \nabla_h \cdot \overline{(K\,\nabla_h q)} - C\bar{q} + \bar{Q}\ .$$

To obtain a numerical solution, equation (6) is approximated by a system of finite differences. The system reported below is found to be very stable. It also gives solutions under simplified conditions that may be compared with analytical solutions, i.e. a Gaussian solution.

The equation is two-dimentional in the horizontal coordinates x and y. A grid system (i,j) in the horizontal plane (x,y) is used to aproximate the equation. The time t is devided into finite time steps denoted by an index k.

$(x)^k_{i,j}$: denotes a quantity x in the grid point i,j at the time step k.

The discrete quantities are supposed to be representative vertical mean values, and the symbol ($^-$) is omitted.

The different parts of equation (6) are approximated by:

$$(7) \quad \frac{\partial(\bar{q})}{\partial t} \simeq \frac{q^{k+1}_{i,j} - q^{k}_{i,j}}{\Delta t}$$

$$(8) \quad \nabla_h \cdot \overline{(\vec{v}_h q)} = \frac{\partial}{\partial x}\overline{(uq)} + \frac{\partial}{\partial y}\overline{(vq)}$$

(9) $$\frac{\partial}{\partial x}\overline{(uq)} \simeq \frac{1}{\Delta x}\left(u^k_{i+\frac{1}{2},j}\, q^k_{i+\ell,j} - u^k_{i-\frac{1}{2}}\, q^k_{i+m,j}\right)$$

$\ell = 0$ when $u_{i+\frac{1}{2},j} > 0$

$\ell = 1$ when $u_{i+\frac{1}{2},j} < 0$

$m = -1$ when $u_{i-\frac{1}{2},j} > 0$

$m = 0$ when $u_{i-\frac{1}{2},j} < 0$

(10) $$\frac{\partial}{\partial y}\overline{(vq)} \simeq \frac{1}{\Delta y}\left(v^k_{i,j+\frac{1}{2}}\, q^k_{i,j+1} - v^k_{i,j-\frac{1}{2}}\, q^k_{i,j+m}\right)$$

$\ell = 0$ when $v_{i,j+\frac{1}{2}} > 0$

$\ell = 1$ when $v_{i,j+\frac{1}{2}} < 0$

$m = -1$ when $v_{i,j-\frac{1}{2}} > 0$

$m = 0$ when $v_{i,j-\frac{1}{2}} < 0$

These differences are based on forward differences in time and up-wind differences in space. This difference system does not give problems at the boundary, provided the pollution that passes into the system across the boundary is known.

In the computations K was supposed to be a constant in the area and $\Delta x = \Delta y = \Delta s$. This gives,

(11) $$\nabla_h \cdot \overline{(K\,\nabla_h q)} \simeq \frac{K}{(\Delta s)^2}\left(q_{i+1,j} + q_{i,j+1} + q_{i-1,j} + q_{i,j-1} - 4.0 q_{i,j}\right)$$

The horizontal velocity was approximated by a stream-function Ψ that was estimated from the wind measurements and given a value in each gridpoint.

(12) $$\vec{v}_h = \vec{k} \times \nabla_h \Psi = -\frac{\partial \Psi}{\partial y}\vec{i} + \frac{\partial \Psi}{\partial x}\vec{j}.$$

In this way the mass budget is automatically taken care of. The vertical flux of pollution is however not simulated in a consistent way, and this has to be taken care of in further developments of the model.

The horizontal velocity in equation (9) and (10) is described by:

$$u_{i+\frac{1}{2},j} = -\frac{1}{4\Delta y}\left(\Psi^k_{i+1,j+1} + \Psi^k_{i,j+1} - \Psi^k_{i+1,j-1} - \Psi^k_{i,j-1}\right)$$

$$u_{i-\frac{1}{2},j} = -\frac{1}{4\Delta y}\left(\Psi^k_{i,j+1} + \Psi^k_{i-1,j+1} - \Psi^k_{i,j-1} - \Psi^k_{i-1,j-1}\right)$$

(13)

$$v_{i,j+\frac{1}{2}} = \frac{1}{4\Delta x}\left(\Psi^k_{i+1,j+1} + \Psi^k_{i+1,j} - \Psi^k_{i-1,j+1} - \Psi^k_{i-1,j}\right)$$

$$v_{i,j-\frac{1}{2}} = \frac{1}{4\Delta x}\left(\Psi^k_{i+1,j} + \Psi^k_{i+1,j-1} - \Psi^k_{i-1,j} - \Psi^k_{i-1,j-1}\right)$$

Finally, the equations (7), (8), (9), (10), (11), (13) are combined in equation (6) to obtain the final difference formulas. If Ψ, K, C, Q, q are known at a time k, these difference equations give the pollution concentration q at a time k+1.

3 APPLICATIONS

During the last part of the winter 1969/70 an investigation of the SO_2 pollution in the air over Oslo was performed. One purpose of this study was to develop a numerical model that could be used to calculate the SO_2 pollution in the air over Oslo whenever wind data are available. This model could then be used to study different air pollution strategies in Oslo.

During pollution episodes in winter time, when cold air becomes stagnant in the Oslo region, the topography and the distribution of heat sources determine the local winds. To study these situations a net of measuring stations were established for wind, temperature and concentration of SO_2 (Figure 1).

It was anticipated that the combustion of oil was the major source of the SO_2 in the air, and source data were obtained from the oil companies. The different companies conducted a survey of the sulphur in all oil that was delivered within each square km in Oslo during the three first months of I970 (Figure 2). The midpoints in each square km form the grid system. It was then assumed that the quantity of oil delivered was equal to the oil burned during the same period. To calculate the fraction of this quantity that was burned each day, the degreeday number was used.

A 24 hour degreeday number denotes how many degrees colder the day's mean temperature is than $17^{\circ}C$, and the oil consumption was assumed to be proportional to this number.

To obtain the hourly variation of the SO_2-emission in an urban area, data given by N C Bown (4) were used (Figure 3).

In order to test the model described, meteorological data and registrations of the SO_2 concentration during the days 26th and 27th February I970 were used. Very high SO_2 concentrations were observed this day. Observations showed that the wind was fairly constant throughout the night and could be approximated by the streamfunction shown in Figure 4. As drainage winds dominated the situation, the windfield is closely related to the topography. The sea was frozen outside the city during February I970, and the city was the only heat source.

It is admitted that the wind stations at Fornebu do not show the relatively strong wind from north-east that the streamfunction indicates. This suggests that vertical air movements are involved over the center of the city, and would imply that some of the SO_2 is transported vertically instead of horizontally out of the lowest layer within the city. As long as the air crossing the upwind boarder of the computing area is clean, this should not change considerations concerning the SO_2 concentration in the center and upwind part of the city.

Between 0900 and 1000 the wind-direction changes 180^{o} at the wind stations around the city, but in the center the wind near the ground was close to zero.

As a first approximation the stream-function in Figure 3 was used during the night, between 09000 and 1000 the wind-velocity was linearly reduced to zero and it remained zero until 1500 houres.

The diffusion coefficient K was estimated to be 10 m^2/sec.

The vertical flux of pollution decreases with increasing vertical temperature gradient. R Reiter and R Sladkovic (5) have concluded from their measurements that the vertical exchange coefficient within an inversion layer is a function of the greatest inverse lapse rate existing in any part of the inversion. The temperature difference between Blindern T_B and Fornebu T_T was used as a rough parameter to describe this greatest inversion lapse rate. The height difference between these two permanent meteorological stations is 84 m.

To take this effect into consideration the parameter C in equation (6) was calculated from the following formula:

$$C = a + b\,(T_B - T_F)$$

$$a = 6.0 \cdot 10^{-4}\ s^{-1} \qquad (14)$$

$$b = -5.0 \cdot 10^{-5}\ s^{-1}\ deg^{-1}$$

The calculation was started on the 26th February at 1500 with zero SO_2 concentration over Oslo. The time increment step was 5 minutes and the calculations were ended on the 27th February at 150 The calculated values of SO_2 are compared with measured values in figures 5 and 6. The result shows that it is possible to calculate with good approximation the content of SO_2 in the air from meteorological measurements and a detailed knowledge of the oil consumption.

4 DISCUSSION

The present model has a time resolution of minutes and a horizontal space resolution of kilometer. The vertical resolution is treated in a very simple way although it is known that vertical diffusion is important in connection with the dispersion of pollution over an urban area. On the other hand, very little is known about the vertical distribution of SO_2 over a city as all the measurements are normally made near the ground. Therefore, the vertical flux of SO_2 is calculated from the mean SO_2 concentration in the lowest layer where all sources exist.

As the model only takes the continuity equation into consideration, it demands detailed measurements of the wind field to be able to consider the concentration of pollution as a kinematic problem. A dynamic approach would explain the local wind field, but would demand a full understanding of,among other things, the effect of all heat sources in the area. As pointed out in the introduction the last approach should be the goal of all studies of the relation between air pollution concentration and meteorological situations.

The kinematic approach is chosen as a first approximation. When the kinematic model works well, as it seems to do in Oslo, the model may be used in constructed meteorological situations to calculate the pollution concentration from a given source distribution. With reasonable estimates of the dispersion effects it may be used in other areas in a similar way. The model does not pretend to explain the dynamics in the local windfield, but it is well suited for air pollution studies when a large amount of measured data exist and are to be used to select the best air pollution control strategy.

The horizontal windfield in Oslo was approximated by a streamfunction, but this is not a necessary approximation. If sufficient data are available,a more realistic windfield may be constructed from the observed winds. This would also give information about the vertical exchange of pollution.

The vertical integrated form of the continuity equation (eq. 6) is well suited to simulate washout by precipitation and thus to calculate the pollution that reaches the ground with it. This has been done earlier by introducing an exponential decay term in the Gaussian plume formula before integrating this with respect to the vertical coordinate.

5 AKNOWLEDGEMENTS

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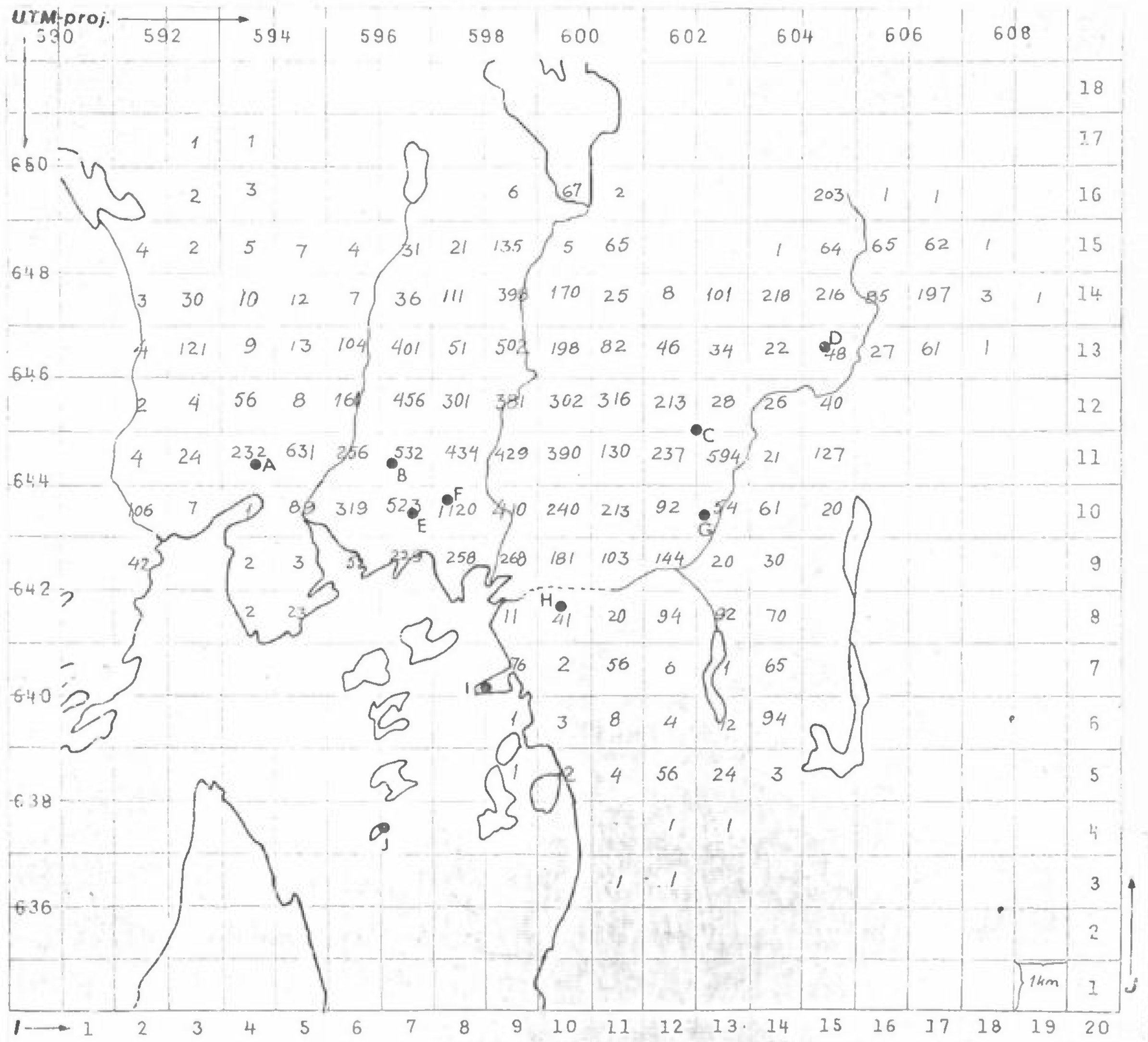

Figure 2: A - Skøyen B - Briskeby C - Økern D - Nyland stasjon E - Haakon VII'sgt F - St Olavsplass G - Bryn skole H - Ekeberg I - Sjursøya J - Husbergøya

The gridsystem and quantum sulphur emitted in Oslo during the first 3 months of 1970.
Unit: 100 kg S/3 months km^2

S: Mean SO_2-concentration every 24 hours

I: Mean SO_2-concentration every $2\frac{1}{2}$ hour

V: Windstation

T: Thermograph

Figure 1 <u>The measuring stations in Oslo. The hatched area denotes the extent of the city</u>

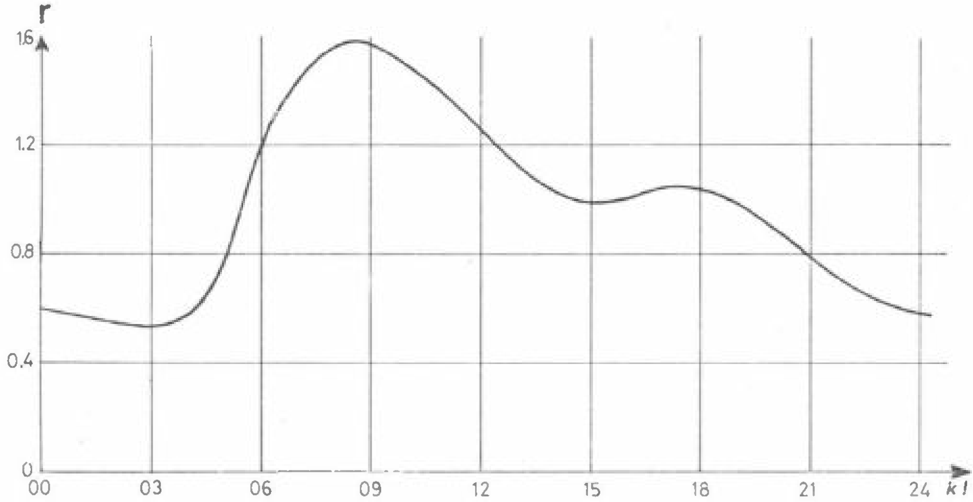

Figure 3: Hourly variation of SO_2- emission. The quotient r between the instantaneous and the mean daily emission of SO_2 is given as a function of the time

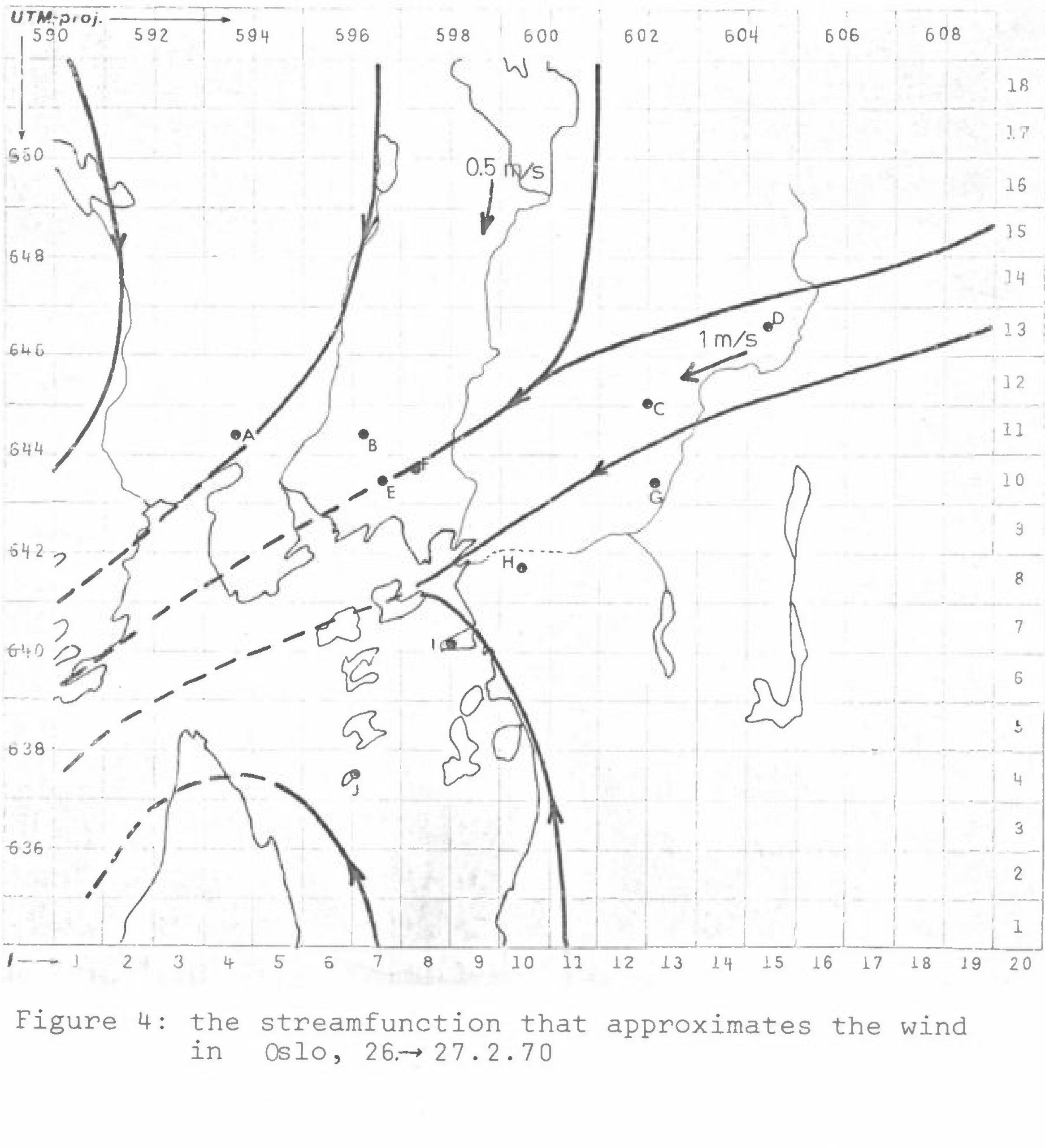

Figure 4: the streamfunction that approximates the wind in Oslo, 26.→ 27.2.70

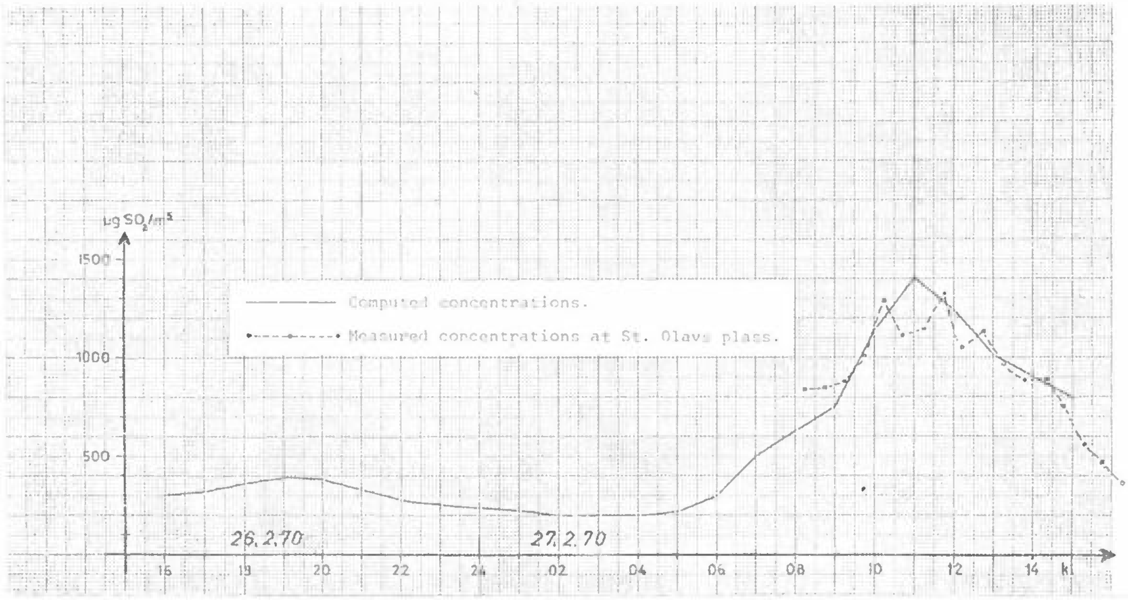

Figure 5: The SO_2-concentration in the gridpoint I=8, J=10 as a function of time

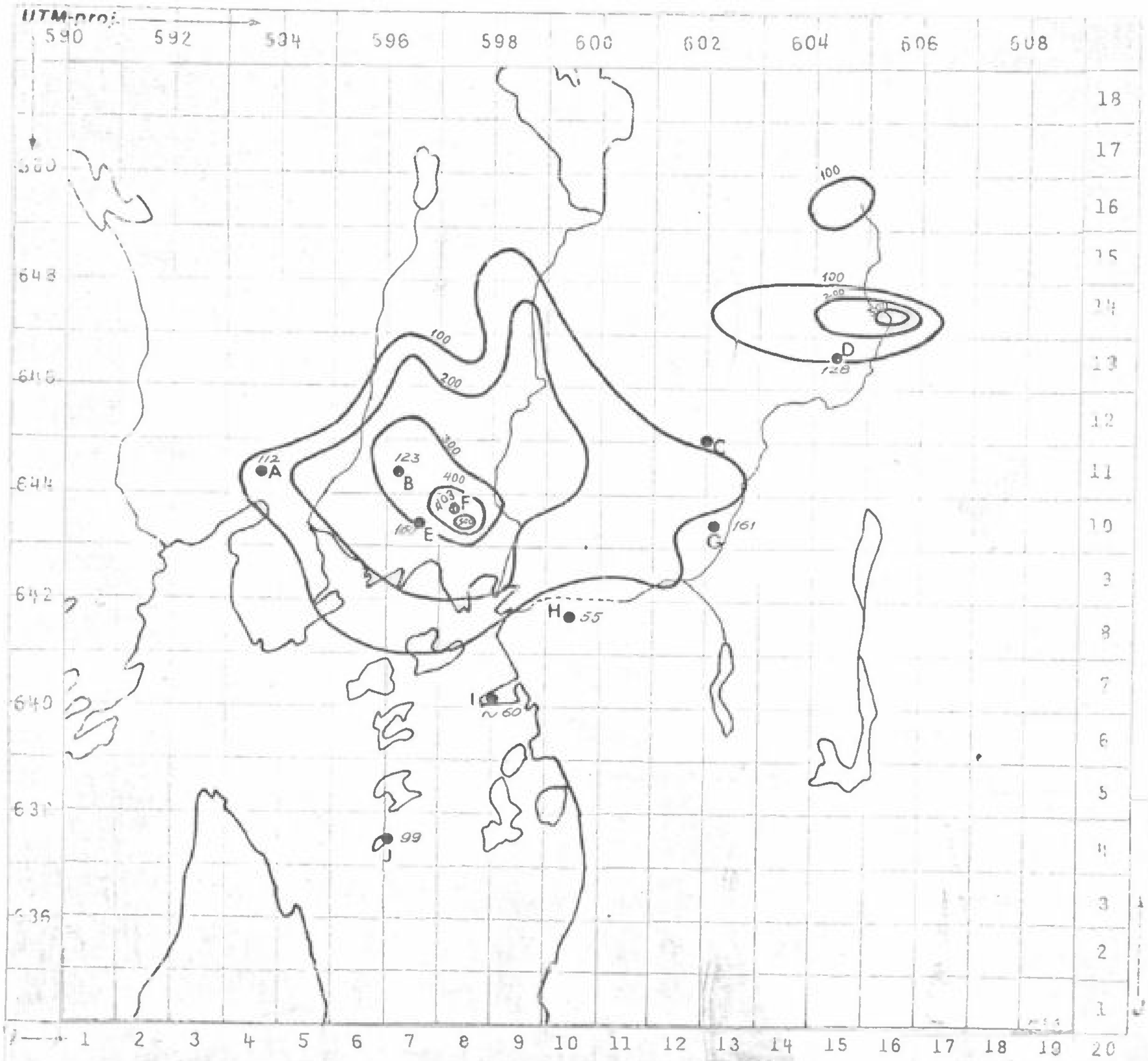

Figure 6: Mean SO_2-concentration from 26.2.70 at 3.00 p.m. → 27.2.70 at 3.00 p.m.