NILU
TEKNISK NOTAT NR. 14/80
REFERENCE: 10180
DATE: JANUARY 1981

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ISBN-82-7247-194-9

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ABSTRACT

The horizontal change in vertical cloud dimensions is evaluated by developing equations for the horizontal derivatives of moments of the vertical concentration dis tribution. The equation for the first moment is used to interpret results of tracer experiments close to highways. The results indicate that the speed of the cars has to be large for the GM-model to be applicable. The EPA-HIWAY model may be applied when the turbulence of the surface boundary layer determines diffusion.

1 INTRODUCTION

When considering vertical mixing of pollution, the ground represents a sink of momentum and a restriction on the vertical scale of eddies. As a consequence, the diffusion conditions are inhomogeneous and the eddy scale is small near the ground, favouring the application of K-theory for vertical diffusion.

In applying dispersion models for air pollution studies, the vertical exchange is often grouped in 4-7 classes depending on wind speed and incoming solar radiation, or on the vertical variation in temperature. Close to a highway the vertical exchange is of primary importance in describing the concentration of pollution emitted from the cars. Systematic deviations between observed and estimated concentrations using the EPA-HIWAY model have been reported (Cadle *et al.*, 1976).

Further results of a finite difference highway model for advection and diffusion of pollution, based on surface layer similarity theory and vehicle wake theory, showed predictions closer to the observations than those of the HIWAY model.

(Eskridge and Hunt, 1979; Eskridge et al., 1979). As a result of vehicle wakes, the vertical diffusion becomes horizontally inhomogeneous. To avoid the labour of numerical solutions of the diffusion equation, in this paper a simplified treatment of the spread from a ground source is adopted.

2 CHANGE IN VERTICAL POLLUTION DISTRIBUTION

Any function of the vertical coordinate, z, may be averaged over the pollution distribution c(x,z):

$$\bar{f}(x) = \frac{\int_{z_1}^{z_2} f(z)c(x,z)dz}{\int_{z_1}^{z_2} c(x,z)dz} \qquad (1)$$

The horizontal variation in the averaged f-function reads:

$$\frac{d\bar{f}(x)}{dx} = \frac{\int_{z_1}^{z_2} (f(z)-\bar{f}(x)) \frac{\partial c(x,z)}{\partial x} dz}{\int_{z_1}^{z_2} c(x,z)dz} \qquad (2)$$

Specifically, different moments of the c-distribution with respect to z may be calculated in this way. The horizontal variation in concentration is given by the diffusion equation simplified to describe the problem under consideration. As an example, the dispersion of pollution from a line source close to the ground is described by the equation:

$$u(x,z) \frac{\partial c(x,z)}{\partial x} = \frac{\partial}{\partial z} (K(x,z) \frac{\partial c(x,z)}{\partial z}) \qquad (3)$$

where: u(x,z) = horizontal wind speed
K(x,z) = coefficient of turbulent exhange

For simple vertical profiles of u and K (both constant with respect to x) the equation may be solved analytically given the boundary conditions. When the analytical procedure is not applicable, numerical methods require a high spatial resolution to give reasonable accuracy. An alternative way is to specify the variation in the moments of the vertical pollution distribution.

Assuming $u(x,z) \neq 0$, Equation (2) reads:

$$\frac{d\bar{f}}{dx} = \frac{\int_{z_1}^{z_2} (f(z) - \bar{f}) \frac{1}{u} \frac{\partial}{\partial z} (K \frac{\partial c}{\partial z}) \, dz}{\int_{z_1}^{z_2} c \, dz} \tag{4}$$

Partial integration of Equation (4) gives:

$$\frac{d\bar{f}}{dx} = \frac{1}{\int_{z_1}^{z_2} c \, dz} \left\{ \Big|_{z_1}^{z_2} \left[\left(f(z) - \bar{f} \right) \frac{K}{u} \frac{\partial c}{\partial z} - cK \frac{\partial}{\partial z} \left(\frac{f(z) - \bar{f}}{u} \right) \right] + \int_{z_1}^{z_2} \frac{\partial}{\partial z} K \frac{\partial}{\partial z} \left(\frac{f(z) - \bar{f}}{u} \right) c \, dz \right\} \tag{5}$$

$$= \frac{1}{\int_{z_1}^{z_2} c \, dz} \left[\Big|_{z_1}^{z_1} \left[\left(f - \bar{f} \right) \frac{K}{u} \frac{\partial c}{\partial z} - c \frac{K}{u} \left(\frac{df}{dz} - \frac{\partial (\ln u)}{\partial z} \left(f - \bar{f} \right) \right] + \right.$$

$$+ \int_{z_1}^{z_2} \left(\frac{\partial}{\partial z} \left(\frac{K}{u} \right) \left[\frac{df}{dz} - \frac{\partial (\ln u)}{\partial z} \left(f - \bar{f} \right) \right] + \frac{K}{u} \left[\frac{d^2 f}{dz^2} - \right. \right.$$

$$\left. \left. - \frac{\partial (\ln u)}{\partial z} \frac{df}{dz} - \frac{\partial^2 (\ln u)}{\partial z^2} \left(f - \bar{f} \right) \right] \right) c \, dz \tag{5}$$

From Equation (5) it is seen that $\frac{K}{u}$, $\frac{d}{dz}(\frac{K}{u})$, and the vertical distribution of ln u determines the atmospheric influence on all moments of the pollution distribution.

These parameters are then describing the growth of the pollution cloud in the atmospheric boundary layer.

Close to the ground:

$$u \frac{\partial c}{\partial x} \rightarrow 0 ,$$

and $z \rightarrow z_o$

where z_o = the roughness length.

The vertical flux of pollution close to the ground is described by deposition processes.

For dispersion calculations, empirically based formulae, considering horizontal variation in the second moment of vertical pollution distribution only (i.e. the Gaussian plume formula) are used. When the atmospheric dispersion conditions are horizontally homogeneous, the accuracy of the results is satisfactory. However, with horizontal change in roughness and/or heatflux from the ground, the vertical diffusivity change, and it is necessary to include this in specific dispersion calculations. Using the first and second moment for dispersion considerations, the following equations may be written.

The first moment:

$$f(z) = z \; ; \qquad \bar{f}(x) = \bar{z}(x) ,$$

The second central moment:

$$f(z) = (z-\bar{z})^2 \; ; \qquad \bar{f}(x) = \overline{(z-\bar{z})^2} = \overline{z^2} - \bar{z}^2 .$$

Using Equation (5) the horizontal derivations of these moments may be written:

$$\frac{d\bar{z}}{dx} = \frac{1}{\int_{z_1}^{z_2} c\, dz} \left[\Big|_{z_1}^{z_2} \left[(z - \bar{z}) \frac{K}{u} \frac{\partial c}{\partial z} - c\, K \frac{\partial}{\partial z} \left(\frac{z - \bar{z}}{u} \right) \right] + \int_{z_1}^{z_2} \frac{\partial}{\partial z} \left[K \frac{\partial}{\partial z} \left(\frac{z - \bar{z}}{u} \right) \right] c\, dz \right] \tag{6}$$

$$\frac{d\overline{z^2}}{dx} = \frac{1}{\int_{z_1}^{z_2} c\, dz} \cdot \left[\Big|_{z_1}^{z_2} \left[(z^2 - \overline{z^2}) \frac{K}{u} \frac{\partial c}{\partial z} - cK \frac{\partial}{\partial z} \left(\frac{z^2 - \overline{z^2}}{u} \right) \right] + \int_{z_1}^{z_2} \frac{\partial}{\partial z} \left[K \frac{\partial}{\partial z} \left(\frac{z^2 - \overline{z^2}}{u} \right) \right] c\, dz \right] \tag{7}$$

3. DISPERSION OF POLLUTION FROM A LINE SOURCE CLOSE TO THE GROUND

Equation (6) is integrated from the ground to a height z_2. Within the area of consideration:

$$c = 0, \text{ and } \frac{\partial c}{\partial z} = 0, \text{ for } z = z_2$$

When dry deposition is small:

$$K \frac{\partial c}{\partial z} \simeq 0, \text{ for } z = z_1$$

According to Equation (6):

$$\frac{d\bar{z}}{dx} = \frac{1}{\int_{z_1}^{z_2} c\, dz} \left[c(z_1) \left(K \left(\frac{\partial \left(\frac{z-\bar{z}}{u}\right)}{\partial z} \right) \right)_{z=z_1} + \int_{z_1}^{z_2} \frac{\partial}{\partial z} \left(K \frac{\partial}{\partial z} \left(\frac{z-\bar{z}}{u} \right) \right) c\, dz \right] \tag{8}$$

$$\frac{d\bar{z}^2}{dx} = \frac{1}{\int_{z_1}^{z_2} c\, dz} \left[c(z_1)\, K \frac{\partial}{\partial z} \left(\frac{z^2-\bar{z}^2}{u} \right)_{z=z_1} + \int_{z_1}^{z_2} \frac{\partial}{\partial z} \left[K \frac{\partial}{\partial z} \left(\frac{z^2-\overline{z^2}}{u} \right) \right] c\, dz \right] \tag{9}$$

The horizontal derivatives of the vertical moment of the c-distribution may be calculated knowing the c-, the K-, and the u-profiles.

The following parameters are defined:

$$c = \bar{c} + \Delta c, \text{ when } z \leq \bar{z}, \text{ and } \int_{z_1}^{z_2} c\,dz = \bar{z}\bar{c}.$$

Equation (8) may then be written:

$$\frac{d\bar{z}}{dx} = \left(\frac{K}{u}\right)_{z=\bar{z}} \frac{1}{\bar{z}} + \frac{1}{\bar{c}\bar{z}} \left(\Delta c K \frac{\partial\left(\frac{z-\bar{z}}{u}\right)}{\partial z} \right)_{z=z_1} + \int_{z_1}^{\bar{z}} \Delta c \frac{\partial}{\partial z} \left(K \frac{\partial}{\partial z} \left(\frac{z-\bar{z}}{u}\right) \right) dz + \int_{\bar{z}}^{z_2} c \frac{\partial}{\partial z} \left(K \frac{\partial \left(\frac{z-\bar{z}}{u}\right)}{\partial z} \right) dz \tag{10}$$

The following transformation of integration variable is made:

$$y = K \frac{\partial\left(\frac{z-\bar{z}}{u}\right)}{\partial z}$$

If it is further assumed that $\frac{dy}{dz} \geq 0$:

$$\frac{d\bar{z}}{dx} = \left(\frac{K}{u}\right)_{z=\bar{z}} \frac{1}{\bar{z}} \frac{c(z_1)}{\bar{c}} + \frac{1}{\bar{z}} \left[\int_{y_1}^{\left(\frac{K}{u}\right)} \frac{\Delta c}{\bar{c}}\, dy + \int_{\left(\frac{K}{u}\right)_{\bar{z}}}^{y_2} \frac{c}{\bar{c}}\, dy \right] \tag{11}$$

From the definition of $\bar{z}$ and Δc it follows that:

$$\int_{z_1}^{\bar{z}} \Delta c\, dz = - \int_{\bar{z}}^{z_2} c\, dz$$

In this way it is seen that the two last integrals have opposite sign and that the sum of the last 2 terms in Equation (11) may be small.

Often an approximation of Equation (11) may be written:

$$\frac{d\bar{z}}{dx} = a \left(\frac{K}{U}\right)_{max} \cdot \frac{1}{\bar{z}} \tag{12}$$

where a = factor of proportionality

$(\frac{K}{U})_{max}$ = the maximum value when $z \leq \bar{z}$.

When K and u are constant with respect to z, the Gaussian plume formula is a solution of Equation (3), and Equations (8) and (9) may be written as:

$$\frac{d\bar{z}}{dx} = \frac{K}{u\bar{z}} \cdot \frac{c(z_1)}{\bar{c}} \qquad (13)$$

$$\frac{d\bar{z}^2}{dx} = \frac{2K}{u} \qquad (14)$$

Considering a Gaussian plume with a standard deviation denoted by σ_z, the following interrelations are found:

$$\frac{c(\bar{z}_1)}{\bar{c}} = \frac{2}{\pi}, \quad \bar{z} = \sqrt{\frac{2}{\pi}}\ \sigma_z, \text{ and } \frac{d\sigma_z}{dx} = \frac{K}{u\sigma_z}$$

These expressions correspond well with the formula previously proposed (Pasquill 1975):

$$\frac{d\bar{z}}{dx} = a \cdot \left(\frac{K}{uz} \right)^{b}_{z=\bar{z}} \qquad (15)$$

According to Equation (13): $a = \frac{c(z_1)}{\bar{c}} = \frac{2}{\pi}$, $b = 1.0$. When using data on K and u profiles defined by similarity theory for the surface layer, it is found that the additional terms in Equation (11) become small for a practical range of thermal stratification. Pasquill indicates that Equation (15) may be used when the Monin-Obukhov length (L) is less than -7 m or larger than 4m.

When an elevated source is considered, z_1 may be selected as the height of the maximum concentration in the plume, when there is no net flux of pollution across this level; the horizontal change in the vertical dispersion parameter may be considered in the same way.

In this way Pasquill's proposal is not restricted to a ground level source.

4 DEFINITION OF DISPERSION PARAMETERS

It is assumed that atmospheric motions have a minor influence on the growth of the wake behind cars. In accordance with Eskeridge et al. (1979 a and b), the vertical exchange coefficient, K_z^w, in the wake is described by the following equation:

$$K_z^w \propto \overline{(w'^2)}^{\frac{1}{2}} \, \ell(s) \tag{16}$$

where w' = fluctuation in vertical wind velocity,
$\ell(s)$ = scale of turbulence,
s = distance behind the car.

$$\overline{w'^2} = w'^2_{max} \cdot F_w(y', z') \tag{17}$$

where y' = horizontal coordinate prependicular to the direction of the road,
z' = vertical coordinate perpendicular to the direction of the road.
F_w = describe spatial variation in w'.

$$F_w(y', z') \approx 1.0, \text{ when } \frac{y'^2+z'^2}{h^2} < \left(\frac{s}{h}\right)^{\frac{1}{2}}$$

$$(\overline{w'^2_{max}})^{\frac{1}{2}} = c_1 U \left(\frac{s}{h}\right)^{-3/4} \tag{18}$$

where c_1 = 0.387 (factor of proportionality),
U = speed of vehicles,
h = average height of cars.

$$\ell(s) = c_2 h \left[\frac{s}{h}\right]^{1/4} \tag{19}$$

where c_2 = 0.53 (factor of proportionality).

According to the Equations (16), (18), and (19), the maximum value of the turbulent diffusivity is:

$$K_z^w = C \cdot h \cdot U \cdot \left(\frac{s}{h}\right)^{\frac{1}{2}} \qquad (20)$$

In a convective atmospheric boundary layer the atmospheric turbulence becomes important close to the highway:

$$\left(\frac{K}{U}\right)_{max} = \left(\frac{K}{U}\right)_{z=\bar{z}}$$

$\frac{K}{U}$ for the atmosphere is described by Monin-Obukhov's similarity theory (see Appendix 1).

During inversion conditions the wake turbulence is dominant close to the highway, and the following expression is found for the cloud dimension:

$$\bar{z} = \left[h \quad 1 + 4\ aC \left(\frac{U}{U_a}\right)^{0.5} \left(\left(\frac{x}{h}\right)^{0.5} - 1\right)\right]^{0.5} \qquad (21)$$

where h = average height of the cars
U = average speed of cars
U_a = wind velocity perpendicular to the highway
x = distance from the edge of a highway

For preliminary calculations the following values are used for the constants:

$$a = \frac{2}{\pi}$$
$$C = c_1 \cdot c_2 = 0.2$$

The following assumption is found reasonable:

$$\bar{z} = h, \text{ when } x = h.$$

The wake-generated turbulence and the atmospheric turbulence may interact in a nonlinear and complex way, encouraging further empirical studies.

As a simplification, the dispersion effect is considered to be an additive effect of atmospheric turbulence and car-generated turbulence:

$$\bar{z} = h\left[1 + 2a\left(\frac{1}{h}\left(\frac{\bar{K}}{U_a}\right)\left(\frac{x}{h} - 1\right) + 2C\left(\frac{U}{U_a}\right)^{0.5}\left(\left(\frac{x}{h}\right)^{0.5} - 1\right)\right]^{0.5} \quad (22)$$

5 INTERPRETATION OF TRACER STUDIES

In order to clarify dispersion conditions close to a highway over a snow-covered plain ($z_o \approx 3$ cm) during inversion situations, SF_6 tracer experiments were carried out. A set of samplers was located out to a distance of 70 m from the edge of a road, as shown in Figure 1. Seven experiments were performed, and the results of one of them were rejected due to variable wind conditions.

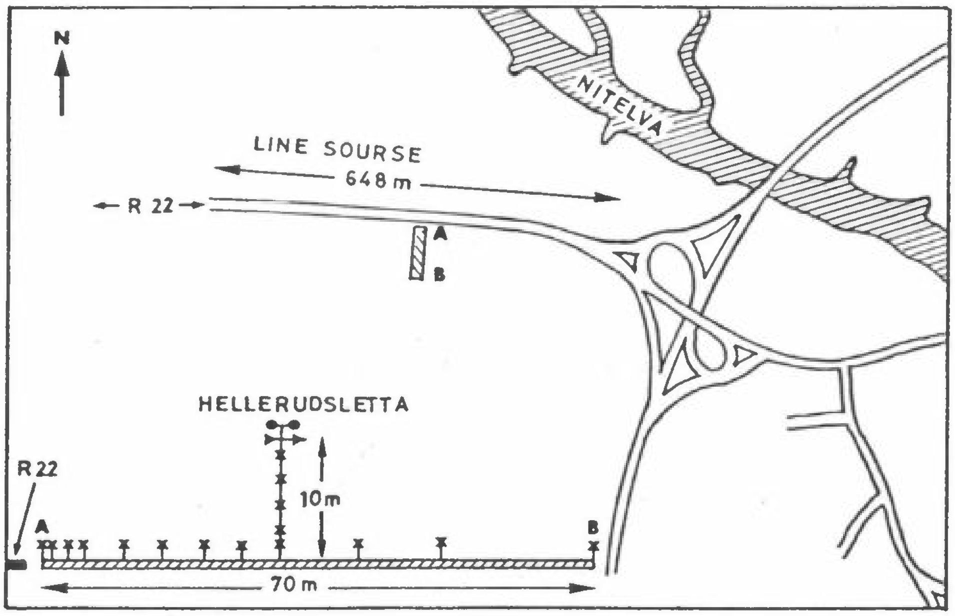

Figure 1: Test area for tracer experiments. Locations of sampling points are shown by x. The automobile emission was simulated by a car driving back and forth along the road and continuously relasing a tracer gas (SF_6).

The vertical distribution of the pollution concentrations was recorded at a distance of 30 m from the edge of the road ($\bar{z}_{30}$). The variation in the vertical dimension of the pollution cloud was also estimated from the fall in ground level SF_6 concentration ($\bar{z}'_{30}$) (see Table 1). Relevant dispersion parameters recorded during the tracer experiments are shown in Table 1.

Table 1: Dispersion parameters during tracer experiments.

Test No.	DATE	U m/s	U_{a10} m/s	$\bar{K}/U_a$ m	$\bar{z}_{30}$ m	$\bar{z}'_{30}$ m
I	13.2.-2	16	1.7	0.09	-	3.5
II	16.2.-1	14.5	0.65	0.003	-	4.4
III	15.3.-1	17.5	2.4	0.08	2.4	2.3
IV	15.3.-2	17.5	1.0	0.08	3.3	3.1
V	16.3.-1	9.7	2.0	0.003	2.3	2.5
VI	16.3.-2	9.7	4.3	0.1	3.0	2.9

U : The speed of the car emitting SF_6

U_{a10}: The wind component perpendicular to the road 10 m above the ground

$\frac{K}{U_a}$: Parameter describing vertical growth of the pollution cloud by atmospheric turbulence (see Equation 22)

$\bar{z}_{30}$: Vertical dimension of the pollution cloud calculated from the measurements on the 10 m mast, 30 m from the edge of the road.

$\bar{z}'_{30}$: Vertical dimension of the pollution cloud deduced from the decay in the concentration close to the ground

Figure 2 shows the vertical dimension of the tracer cloud 30 m downwind from the road ($\bar{z}_{30}$) for each of the tracer experiments. The values are shown as a function of wind speed perpendicular to the road. The parameters that are relevant to dispersion are shown in Table 1, together with observed values of the vertical dimension ($\bar{z}_{30}$). Both atmospheric turbulence and wake effects from the car contribute to the vertical growth of the cloud as shown on the figure by a solid line and a broken line, respectively. According to Equation (22) large variation in the two effects may occur from one experiment to the next.

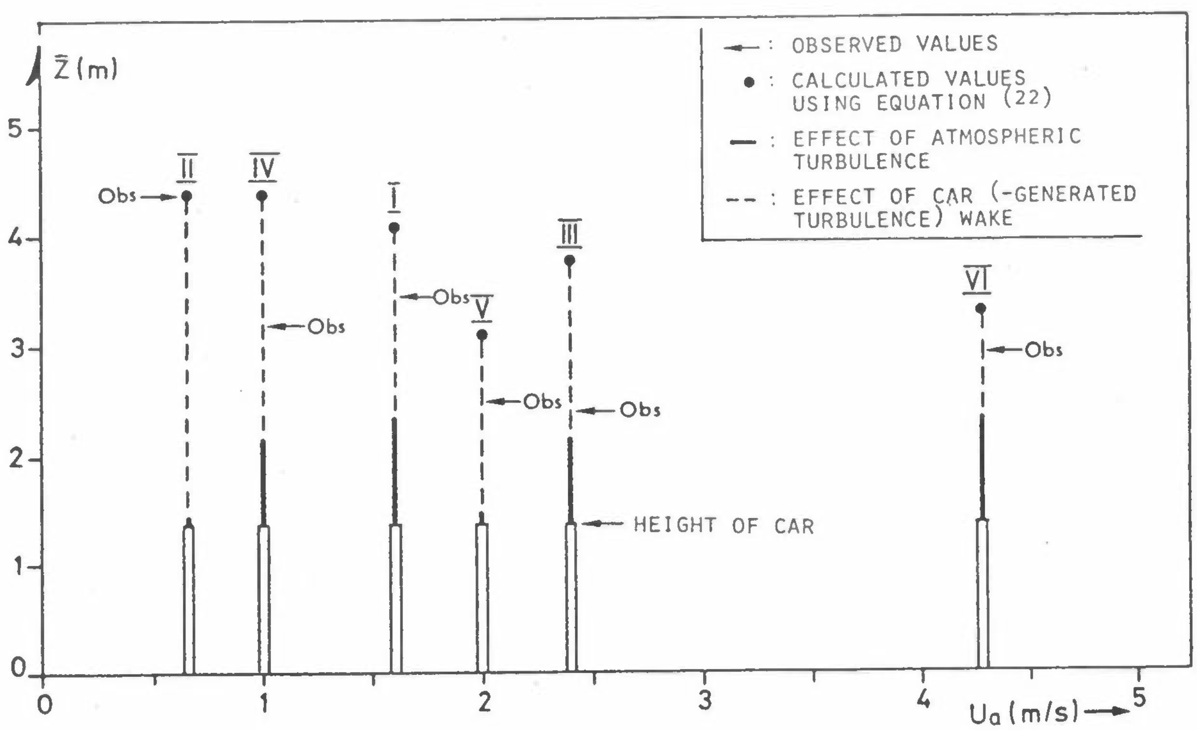

Figure 2: Vertical dimension ($\bar{z}$) of tracer cloud 30 m from the edge of the road given as a function of wind speed perpendicular to the road. The dimension of the cloud is assumed to be the height of the car (1.4 m) at the edge of the road. For each of the experiments the observed values (↑) and the caluculated values (o) are shown. For the calculated values the effect of atmospheric turbulence is shown by a solid line (——) and the effect of car-generated turbulence by a broken line (----).

It is seen that the atmospheric effects caused very little diffusion in experiments 2 and 5. The dispersion caused by wake effects became large especially when the wind speed was low.

Compared with other models, our results indicate a less effective vertical dispersion than that predicted by the GM model when the wind speed is higher than 2 m/s, perhaps because our car was driven at a lower speed than the GM test cars. By selecting the proper class of stability, our results show more effective vertical diffusion than that predicted EPA-HIWAY model, since the wake effect contributed significantly in all our experiments. However, the EPA-HIWAY model gives a realistic description of the atmospheric turbulence effect and should apply when the wake effect is weak.

In Figure 2 both observed and calculated values show a tendency for the vertical dimension to become smaller with increasing wind speed. This is a result mainly of the wake effect on dispersion that is large when the wind speed is small.

The stability classes are traditionally used to classify the increase in vertical dimensions of the pollution cloud. Our results indicate, however, that the horizontal wind speed should also be taken into account according to the theory presented in the previous sections.

The empirically verified by GM model should apply when the speed of the cars is about 80 km per hour. As reported by Rao and Keenan (1980), the vertical dimension of the pollution cloud at the nearest roadside receptor is a function of cross-road wind speed. In Figure 3, calculated values from Equation (21) are compared with the observed values reported by Rao and Keenan.

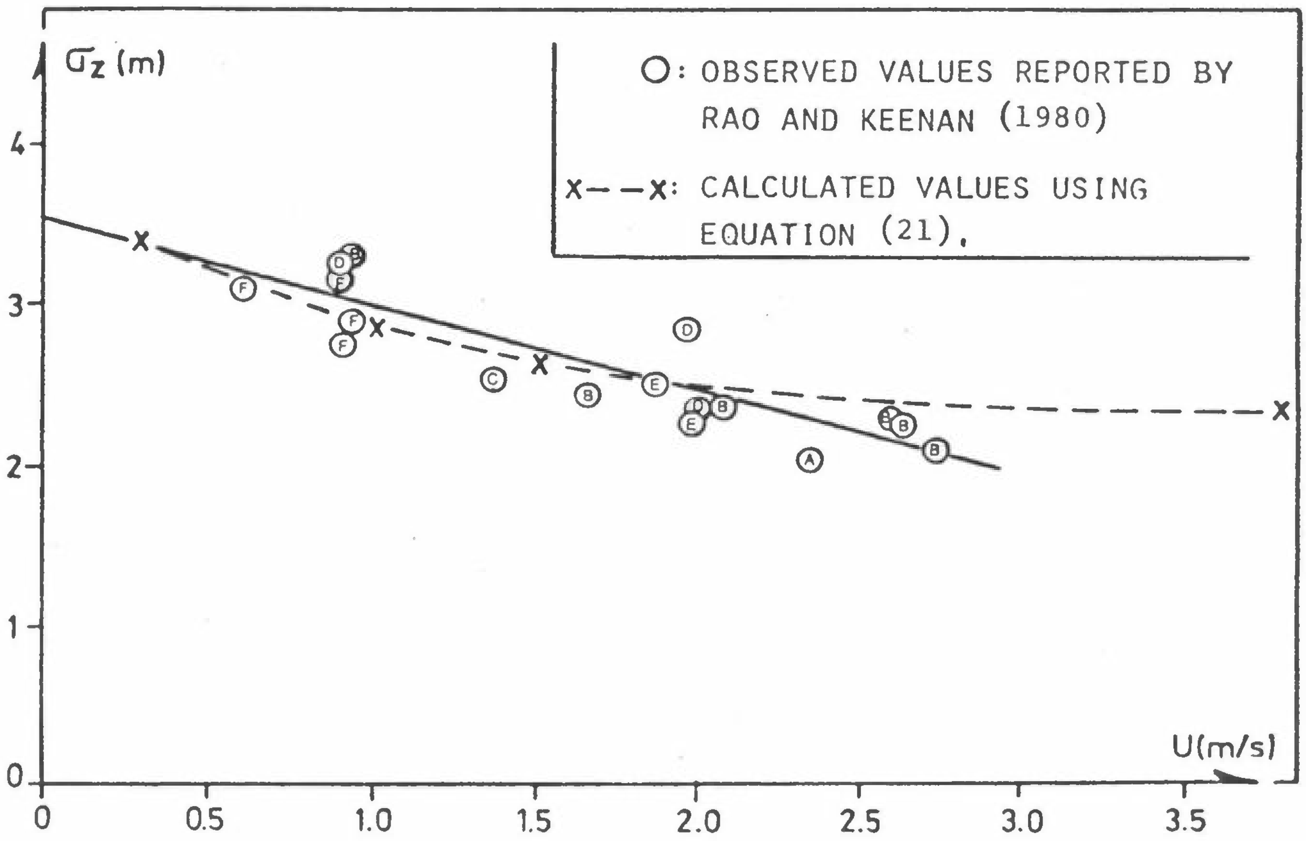

Figure 3: The standard deviation, σ_z, near the roadside (∿ 4 m from the edge of road) as a function of cross-road wind speed.

ACKNOWLEDGEMENTS

The author wish to thank colleagens and specially Steinar Larssen and Val Vitols for good cooperation and helpful comments.

APPENDIX: EVALUATION OF METEOROLOGICAL MEASUREMENTS BY SIMILARITY THEORY

The following terminology is used:

z = vertical coordinate
z_o = roughness parameter
L = Monin-Obukov length
c_p = specific heat of air at constant pressure
ρ = density of air
T_o = temperature at the ground
$\rho u^2_* = \rho\, \overline{-u'w'}$ vertical flux of momentum
$u_* T_* = \overline{w'\Theta'} = \frac{H_o}{c_p \rho}$ vertical flux of sensible heat
κ = von Karman's constant
ϕ_m = normalized wind gradient $= (\kappa z/u_*)\ (\frac{du}{\partial z})$
ϕ_h = normalized temperature gradient $= ((\kappa z/\Theta_*)\ \frac{\partial \Theta}{\partial z})$

To determine the mean air flow close to the ground, the following "forces" are important:

1. The horizontal stress $(\frac{\partial \tau}{\partial z})$ has to be known for the horizontal equation of motion, and follows from:

$$\tau = \rho u_*^2$$

2. In a stratified atmosphere, a buoyancy force influences the vertical motion. It may further be shown that this force is proportional to the Vaisälä-Brundt frequency for small perturbations:

$$\nu_s^2 = \frac{g}{T}\ (\frac{dT}{dz} + \frac{g}{c_p})$$

3. For the thermodynamic energy equation, the vertical flux of sensible heat (H_o) has to be known:

$$\overline{w'\Theta'} = u_* T_*$$

Monin-Obukhov similarity theory assumes that the vertical flux of heat and momentum, together with the height z and the Vaisälä-Brundt frequency ν_s, constitute the important parameters.

They defined a length parameter, L, and a temperature parameter, θ_*:

$$L \overset{\text{def}}{=} - \frac{u_*^3 \, T}{\kappa g \, u_* \theta_*}, \text{ and } \Theta_* \overset{\text{def}}{=} \frac{\overline{w'\Theta'}}{u_*}$$

Two nondimensional products may be defined, and the π theorem is used to describe the vertical gradients of wind speed, $\frac{\partial u}{\partial z}$, and potential temperature, $\frac{\partial \Theta}{\partial z}$:

$$\frac{\kappa z}{u_*} \frac{du}{dz} = \phi_m(\frac{z}{L}) \qquad \text{(A1)}$$

$$\frac{\kappa z}{\Theta_*} \frac{d\theta}{dz} = \phi_h(\frac{z}{L}) \qquad \text{(A2)}$$

The use of Businger's empirically determined universal functions (Businger 1973), has been proposed (Busch et al., 1976) when these equations are integrated. With $\zeta = (z + z_o) \cdot L^{-1}$, the following results may then be written:

When $\zeta < 0$:

$$\frac{u}{u_*} = \frac{1}{\kappa} (\ln \frac{z}{z_o} - \Psi_1) \qquad \text{(A3)}$$

$$\Psi_1 = 2 \ln ((1+x)/2) + \ln ((1+x^2)/2) - 2 \tan^{-1} (x+\pi/2)$$

$$x = (1 - 15 \zeta)^{1/4} = \phi_m^{-1}$$

$$\frac{\Theta - \theta_o}{\theta_*} = 0.74 (\ln \frac{z}{z_o} - \psi_2) \qquad \text{(A4)}$$

$$\psi_2 = \ln ((1+y)/2)$$

$$y = (1-9\zeta)^{\frac{1}{2}} = 0.74 \, \Phi_h^{-1}$$

When $\zeta > 0$:

$$\frac{u}{u_*} = \frac{1}{\kappa} \left(\ln \frac{z}{z_o} + 4.7\ \zeta\right) \tag{A5}$$

$$\frac{\bar{\theta} - \theta_o}{\theta_*} = 0.74\ \ln \frac{z}{z_o} + 4.7\ \zeta \tag{A6}$$

$$\Phi_m = 1 + 4.7\ \zeta$$

$$\Phi_h = 0.74 + 4.7\ \zeta$$

With measurements of wind and temperature at two levels as inputs, Equations A3 and A4, or A5 and A6, may be solved by iteration to find u_*, Θ_*, and accordingly L. Only a few iteration steps are necessary.

The turbulent exchange of pollution (K_z) is estimated by using the turbulent exchange of heat, K_h, and the following formula is used:

$$K_z = K_h = \kappa u_* z/\phi_h(z/L) \tag{A7}$$

This parameter may be used to estimate dispersion of pollution in the surface boundary layer.

When the horizontal scale is large compared to the vertical scale, the three "forces" above (points 1.-3.) may be considered the dominant terms in the hydrodynamic equations near the ground, and local similarity theory may be used. In more complex, inhomogeneous situations other effects may influence the vertical structure given in Equations A1 and A2, or a quasi-stationary situation may not exist.

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POSTBOKS 130, 2001 LILLESTRØM
ELVEGT. 52.

TLF. (02) 71 41 70

RAPPORTTYPE TN	RAPPORTNR. 14/80	ISBN--82-7247-194-9
DATO 21.1.1981	ANSV.SIGN. B. Ottar	ANT.SIDER 19
TITTEL Forenklet betraktning av vertikal-spredning ved inhomogene atmosfæriske forhold.		PROSJEKTLEDER K.E. Grønskei NILU PROSJEKT NR 10180
FORFATTER(E) K.E. Grønskei		TILGJENGELIGHET ** A OPPDRAGSGIVERS REF.
OPPDRAGSGIVER Norsk Institutt for Luftforskning		
3 STIKKORD (å maks.20 anslag) Diffusjon	Highway-modell	Moment method

REFERAT (maks. 300 anslag, 5-10 linjer)

Vertikalspredning vurderes ved å utvikle ligninger for horisontalderiverte momenter av vertikale konsentrasjons-fordelinger. Ligningene er benyttet til å tolke resultater av sporstoffundersøkelser ved veibaner.

TITLE Simplified treatment of vertical diffusion under inhomogeneous atmospheric conditions.

ABSTRACT (max. 300 characters, 5-10 lines)

The horizontal change in vertical cloud dimension is evaluated by developing equations for the horizontal derivations of moments of the vertical concentration deistribution. The equation for the first moment is used to interpret results of tracer experiments close to highways.

**Kategorier: Åpen - kan bestilles fra NILU A
Må bestilles gjennom oppdragsgiver B
Kan ikke utleveres C