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PLUME RISE CALCULATIONS

by

B. Sivertsen

NORWEGIAN INSTITUTE FOR AIR RESEARCH
P.O. Box 115, 2007 KJELLER
NORWAY

CONTENTS

A = correction factor for stability : 2.65, 1.08, .68 for unstable, neutral and stable, respectively

C = ground level concentration at plume centerline (g/m^3)

d = inner stack diameter (m)

F = flux of buoyant force carried by the stack gases divided by π and the atmospheric density (m^4/s^3)

g = acceleration of gravity (m/s^2)

H = Plume centerline height above ground level (m)

h_s = stack height (m)

Δh = plume rise height above the stack (m)

$\dot{Q}$ = emission rate of gaseous effluent (g/s)

Q_H = heat emission (cal/s)

Q_{MW} = heat emission (MW)

s = stability parameter (s^{-2})

T = mean temperature of ambient air ($^{\circ}K$)

T_s = stack gas temperature ($^{\circ}K$)

ΔT = $T_s - T$

U = average wind speed at stack level (m/s)

U_P = average wind speed at height of plume (m/s)

U_m = average wind speed between stack top and plume top (m/s)

W = exit gas velocity (m/s)

x = distance downwind from the stack (m)

x^* = distance at which atmospheric turbulence begins to dominate growth of plume (m)

z = height above the stack (m)

$\frac{\partial \theta}{\partial z}$ = vertical potential temperature gradient of the atmosphere (deg/m)

σ_y, σ_z = standard deviation of respective crosswind and vertical normally distributed concentration of plume effluent for a specific downwind distance and atmospheric stability (m)

1 INTRODUCTION

The following comments on plume rise calculations were first prepared as a preliminary draft during my one year leave of absence from NILU at Burns & Roe Inc., (Long Island, New York). The importance of a proper choice of plume rise, when designing stack heights is demonstrated. It should be emphasized that there does not exist one unique formula which applies to all stacks, all conditions and all sites. The formula to use must be selected based on a proper analysis of the type of emission (size, heat, exit velocity etc). Even then, under the best conditions, an uncertainty by a factor of two in estimates of the plume rise is likely on any one occasion, because of the natural variability of the atmosphere (14).

2 GROUND LEVEL CONCENTRATION

To calculate maximum ground level concentrations of air pollutants for comparison with Air Quality Standards, or for determining minimum stack heights, the following formula of gas dispersion is used:

$$C = \frac{\dot{Q}}{\pi \sigma_y \sigma_z u} \exp\left(-\frac{H^2}{2\sigma_z^2}\right) \qquad \text{(eq. 1)}$$

The parameter H denotes the plume (centerline) height above the ground and can be expressed as:

$$H = h_s + \Delta h - k \cdot h_{el} \qquad \text{(eq. 2)}$$

where h_s is the physical stack height, Δh is the plume rise due to buoyancy and exit gas velocity and h_{el} is "ground level" elevation relative to the stack base level, or the mean height of the buildings in the area around the stack. k is a constant factor varying from 0 to +1.0 dependent on the stack height compared to h_{el}, or the distance to and the shape of the topographical irregularities.

3 PLUME RISE FORMULAS

When calculating minimum stack heights or air quality for large power plants, the results obtained are sensitive to the plume rise Δh, and it is important to choose the right formula for each specific case. More than 30 plume rise formulas appear in the literature today, and new formulas are presented each year.

The Holland (Oak Ridge) formula from 1953 is one of the oldest plume rise formulas used today.

$$\Delta h_H = 1.5\ d \cdot \frac{W}{U} + 4.0 \cdot 10^{-5}\ \frac{Q_H}{U} \qquad \text{(eq. 3)}$$

The Holland formula is empirical. It is based on relatively small sources and observations of plume rise fairly close to the stack. It thus greatly underestimates the final plume rise at power plants (3), (16), (17).

Stümke introduced in 1962, a correction factor of 2.92 to the Holland formula. This correction was still based on moderate sized plants (see Appendix I), and Slade (16) points out that this formula still underestimates the plume rise for very large plants. This formula will, however, overestimate plume rise for industrial sources and small to moderate sized plants.

In 1963, Stümke presented an empirical modification of the Holland formula optimizing his adjustable parameters for best fit data:

$$\Delta h_S = \frac{1}{U} (1.5 \cdot W \cdot d + 65.0 \cdot d^{3/2} (\frac{\Delta T}{T_S})^{1/4}) \qquad \text{(eq. 4)}$$

This is a good formula for industrial sources and medium sized plants. The formula underestimates the plume rise, but Briggs (3) concludes that among the empirical formulas, Stümke is one of the best ones. This statement was based on applying it to 16 different sources during near neutral conditions.

An especially simple plume rise formula was derived by the CONCAWE working group (7). Using several hundred observations in Western Europe this group developed a regression formula based on the assumption that plume rise depends mainly on heat emission (Q_H) and wind speed (U). The observations were, however, taken only from 8 stacks and the data fall into a small range of Q_H and U. The CONCAWE formula was tested on full scale large electric generating stations of TVA (17) and it showed good agreement with the actual plume rise. The simplified CONCAWE formula is:

$$\Delta h_C = .175 \left[Q_H^{\frac{1}{2}} U^{-3/4} \right] \qquad \text{(eq. 5)}$$

Bringfelt (6) also arrived at a simple expression for the plume rise based on about 70 measurements of smoke plume trajectories at industrial chimneys. He assumed that the plume rise at a fixed distance is proportional to U^{-1}, and uniquely related to the heat emission Q_{MW} (in megawatts). A regression analysis for neutral stability gave the following equations:

Distance from source: 250 m: $\Delta h_B = 103 \cdot Q_{MW}^{0.39} \cdot U^{-1}$

" " " 500 m: $\Delta h_B = 167 \cdot Q_{MW}^{0.36} \cdot U^{-1}$

" " " 1000 m: $\Delta h_B = 224 \cdot Q_{MW}^{0.34} \cdot U^{-1}$

(eq. 6)

A survey of 11 plume rise formulas was presented by Carson and Moses in 1969 (13). This survey was based on 711 observations from 9 stacks, with heat emissions ranging from 0.06 MW (Argonne) to 120 MW (Paradise Plant). More than 80% of the observations were from medium sized and small power plants. Carson and Moses concluded that based on the best fit and ease of computation the preferred plume rise equation was on the form:

$$\Delta h_{CM} = \frac{A}{U}\left[-.029\ w \cdot d + 5.35\ (10^{-3} \cdot Q_H)^{\frac{1}{2}}\right] \quad \text{(eq. 7)}$$

The plume rise formulas given so far are all empirical formulas based on a limited number of observations of actual plume rises from a limited number of different stacks.

A simple theoretical model is later developed, in which a bent-over plume is emitted from a point source of conserved buoyancy and the plume radius increases proportional to the height of the rise. From this theory the well-known "2/3-law" of the plume rise is developed. This gives a plume rise proportional to $x^{2/3}$, and it is seen to give better agreement than any of the empirical formulas (2), (4).

Briggs (5) concludes based on data from several investigators, that for buoyancy-dominated rise in unstratified ambients or for the early part of the rise in stratified ambients, the 2/3-law yields on the form:

$$\Delta h = C_1 F^{1/3} U_P^{-1} \cdot x^{2/3} \qquad \text{(eq. 8)}$$

where: $F = gw (d/2)^2 \Delta T/T_S$ (eq. 9)

C_1 is found from different observed data to range from 1.2 to 2.6. The bulk of the data support values of C_1 ranging from 1.6 to 1.8, and the value $C_1 = 1.6$ is recommended to be slightly on the conservative side (5).

During neutral asmospheric conditions this equation is valid up to a distance $x = x^*$ where

$$x^* = 2.16 F^{2/5} h_S^{3/5} \qquad \text{(eq. 10)}$$

Beyond this distance the plume centerline is more accurately described by:

$$\Delta h = \frac{C_1 F^{1/3}}{U_P} x^{*2/3} \left[\frac{2}{5} + \frac{16}{25} \frac{x}{x^*} + \frac{11}{5} \left(\frac{x}{x^*}\right)^2 \right] \left(1 + \frac{4}{5}\frac{x}{x^*}\right)^{-2}$$

(eq. 11)

The plume rise on great distances is, however, very sensitive to the turbulence characteristics of the atmosphere, the terrain features, roughness, etc. and large deviations might occur from one case to another. For fossil-fuel plants with heat emission of 20 MW or more, a good working approximation is given by (4):

$$\Delta h = \frac{1.6 F^{1/3}}{U_P} x^{2/3} \quad (\text{for } x < 10\ h_s) \qquad (\text{eq. } 12)$$

$$\Delta h = \frac{1.6 F^{1/3}}{U_P} (10\ h_s)^{2/3} \quad (x > 10\ h_s) \qquad (\text{eq. } 13)$$

For other sources, the final plume rise is given by:

$$\Delta h = \frac{1.6 F^{1/3}}{U_P} (3\ x^*)^{2/3} \qquad (\text{eq. } 14)$$

These equations also appear to be valid in unstable conditions, but the scatter about the mean centerline is greater. These formulas are recommended to use by Slade (16, page 198), Manier (10, page 159), Briggs (2, page 57), Fay et al (8, page 396) and Altomare (1, page 11).

If the stack height (h_s) is unknown, or the problem is to find the stack height, Altomare (1) has given an alternate way of calculating final plume rise:

$$\Delta h = \frac{1.6 F^{1/3}}{U_P} (3.5\ x^*)^{2/3} \qquad (\text{eq. } 15)$$

where: $x^* = 14\ F^{5/8}$ for $F < 55\ m^4/sec^3$

$$x^* = 34\ F^{2/5} \text{ for } F > 55\ m^4/sec^3 \qquad (\text{eq. } 16)$$

This latter approach is also suggested in a EPA recommendation for dispersion estimates dated May 1973 (18).

During stable atmospheric conditions the prediction of the final rise of buoyancy-dominated plumes in a constant potential density gradient is given by Briggs:

$$\Delta h = C_2 \left(\frac{F}{U_p s}\right)^{1/3} \qquad \text{(eq. 17)}$$

where the stability parameter $s = \frac{g}{T} \left(\frac{\partial \theta}{\partial z}\right)$

Plume rise observations from 18 different sources indicate a value of C_2 ranging from 1.8 to 3.1. Briggs (5) proposes $C_2 = 2.4$ to be a good average value, slightly on the conservative side. The above expression for final plume rise in a stable atmosphere is recommended by most of the authors in the attached list of references. Turner (18) also gives the value of $\frac{\partial \theta}{\partial z}$ for the Pasquill stability classes:

For class E: $\frac{\partial \theta}{\partial z} = 0.02$, for class F: $\frac{\partial \theta}{\partial z} = 0.035$.

For calm conditions Briggs suggests a final plume rise given by:

$$\Delta h = 5\ F^{1/4} \cdot s^{-3/8} \qquad \text{(eq. 18)}$$

For low wind speeds the smaller of the two Δh's should be used (from eq. 17 or 18).

Recently Moore (12) presented a comparison of trajectories of rising plumes with theoretical empirical models using plume rise data from USA, Great Britain and Sweden. His goal was to produce an expression which gave the best possible estimate of the plume height, for all reasonable emissions and meteorological conditions.

In the conclusion of his paper it is stated that:
"It is unlikely that a significant better expression than

$$\Delta h = \frac{A}{U_m} \; Q_{MW}^{1/4} \cdot x^{*\,3/4} \qquad \text{(eq. 19)}$$

can be found to represent the trajectories of boiler plant plumes with $h_s \geq 120$ m over the distance range 400 - 2500 m downwind".

A rather complicated method for calculating the "final plume rise" is indicated. There is, however, little sacrifice in accuracy by replacing $Ax^{*\,3/4}$ with Lucas' expression of 1967 (9) $Ax^{*\,3/4} = 275 + 2\,h_s$ for "average meteorological conditions and $Ax^{*\,3/4} = 60 + 5h_s$ for unstable or adiabatic conditions. For plumes with much higher efflux velocities, or very different initial density from boiler plants, some small modifications to the expression are required.

A recent TVA investigation of plume rise (11) demonstrates how the rate of plume rise with downwind distance depends on the atmospheric stability. Based on more than 1100 photographs of the TVA power plants, the following expressions for plume rise are developed:

Neutral conditions: $(-1.7 \cdot 10^{-3} < \frac{\partial \theta}{\partial z} \leq 1.6 \cdot 10^{-3})$

for x up to 3000 m:

$$\Delta h = 2.5F^{1/3}\, x^{0.56}\, U_m^{-1} \qquad \text{(eq. 20)}$$

Moderately stable conditions: $(1.6 \cdot 10^{-3} < \frac{\partial\theta}{\partial z} \leq 7.0\ 10^{-3})$

for x up to 2800 m:

$$\Delta h = 3.75 \cdot F^{1/3}\ x^{0.49}\ U_m^{-1} \qquad \text{(eq. 21)}$$

Very stable conditions: $(7.0 \cdot 10^{-3} < \frac{\partial\theta}{\partial z} \leq 1.87 \cdot 10^{-2})$

for x up to 1960 m:

$$\Delta h = 13.8\ F^{1/3}\ x^{0.26}\ U_m^{-1} \qquad \text{(eq. 22)}$$

The TVA power plants consist of rather large coal fired units with stack heights between 152 and 244 m. At a selected downwind distance of 1824 m from the stacks an equation of plume rise as a function of the stability (all stability conditions) was developed for the TVA-plants:

$$\Delta h\ (1824\ m) = 173\ F^{1/3} \cdot U_m^{-1} \cdot \exp(-64\ \frac{\partial\theta}{\partial z}) \qquad \text{(eq. 23)}$$

4 CONCLUSIONS AND RECOMMENDATIONS

Several plume rise formulas exist. Since most of the formulas are based on empirical data, the functional forms may vary from one application to another. The application of one formula should not be extended to ranges outside those of the field data on which it is based.

The above survey of some plume rise formulas leads to the following recommendations:

For small heat emission (< 1 MW) the Holland type plume rise formula (or Stümke) seem to be appropriate for calculating the final plume rise.

For moderate sized power plants (heat emission: 1-30 MW) and industrial sources the Stümke formula (eq. 4) seems to be a good working formula, which fits observed data well.

For large power plants (heat emission > 30 MW) with high stacks and buoyancy-dominated plume rise the Briggs equations (eq. 13 and 14) are recommended for calculating the plume rise. These equations should not be applied to cases where there is substantial momentum rise in relation to buoyancy rise. The Stümke formula is then preferred.

When the aim is to calculate the physical stack height for a new plant, an approach given by Altomare (eq. 15, 16) is recommended. This approach should, however, only be applied to sources with large heat emissions and warm moist plumes (cooling towers).

During stable atmospheric conditions the Briggs stable air equation for plume rise (eq. 17) is recommended for all sources.

5 REFERENCES

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APPENDIX I

PLANT SIZE RELATED TO HEAT EMISSION

The expressions: small, moderate sized and large power plants are used in the text. These expressions are usually, when plume rise calculations are considered, defined by the heat emission from the plant stack (from each stack). When reviewing the literature, the terms are defined slightly different. A summary of the different sources gave the following definitions:

A small plant is a plant with stack heat emission

$Q_{MW} < 1$ MW ($Q_H < 2 \cdot 10^5$ cal/s)

At a medium size plant: Q_{MW} is between 1 and 30 MW

(Q_H : $2 \cdot 10^5 - 7 \cdot 10^6$ cal/s)

At a large plant:

$Q_{MW} > 30$ MW ($Q_H > 7 \cdot 10^6$ cal/s)

APPENDIX II

PLUME RISE FOR DIFFERENT SOURCES

The final plume rise at 7 different sources is listed in the following table for 8 different plume rise formulas. The calculations are performed for a wind speed of 4 m/s, which is assumed to be constant with height, and for near neutral atmospheric conditions.

	Source no.						
	I	II	III	IV	V	VI	VII
Heat emission Q_{MW} (MW)	4	5	13	33	42	61	64
Airflow (Nm^3/s)	300	17,2	96	160	168	389	312
Gas temp. T_s (^oK)	293	583	383	440	473	403	440
Air temp. T (^oK)	283	283	283	283	283	283	283
Gas ex. vel. W (m/s)	25,0	13,0	10,0	13,8	10,0	15,0	19,1
Stack diam. d(m)	4,0	1,8	3,0	4,9	6,0	6,9	5,8
Stack height h_s (m)	100	60	50	72	100	140	200
PLUME RISE FORMULA:	FINAL PLUME RISE (m)						
Holland	47	22	41	104	122	184	194
Stümke	93	41	72	161	213	256	217
Moses-Carson	43	52	79	127	144	173	178
Concawe	52	63	102	177	204	254	261
Briggs (10 hs)	75	75	114	215	290	412	532
Briggs/Altomare	79	101	167	297	342	430	442
Bringfelt (1000 m)	89	100	133	184	200	227	231
Moore	(167)	(147)	(177)	(250)	(301)	387	475

APPENDIX III

SAMPLE CALCULATION

A sample calculation is performed for two different oil fired power plants.

		Plant I	Plant II
Plant Data	Power (MW)	195	450
	Stack i.d. (m)	4.88	5.80
	Gas velocity (m/s)	13.8	19.05
	Stack gas temp. (^oK)	440	440
	Stack height (m)	72	200
Plume Rise (m) (u_{100} = 5 m/s)	Holland formula	84	125
	Holland w/Stümke	246	
	Stümke	137	151
	Concawe	140	163
	Carson & Moses	106	117
	Briggs Trans. (10 h_s)	163	350
	Briggs Trans. (3.5x)	213	327
	Briggs stable	124	177
	Bringfelt (500 m)	120	140
	Bringfelt (1000 m)	150	170
	Moore	(182)	285

The calculations above are based on a mean wind velocity at 100 meter level above ground (u_{100}) = 5 m/s.

U_M is obtained by assuming a potential wind profile on the form

$$u_z = u_{100} \left(\frac{z}{100}\right)^{\frac{1}{4}}$$

which gives approximately

$$U_m = \frac{u_{100}}{6.32} \left(h_s^{1/4} + (h_s + \Delta h)^{1/4}\right)$$

As seen from the table the Holland formula differs from the Briggs transitional by a factor 2.6 or 2.8 for the 450 MW plant (depending on which final form of Briggs is used), and by a factor 1.9 or 2.5 for the 195 MW plant.

If searching for the maximum ground level concentrations assuming the dispersion coefficients are simple functions of the distance x:

$$\sigma_y = c_y \, x^p$$

$$\sigma_z = c_z \, x^q$$

Assuming neutral atmospheric conditions, and using the Brookhaven dispersion coefficients for 1-hr. average concentrations we get:

$c_y = .32$ $\qquad$ $p = .86$

$c_z = .216$ $\qquad$ $q = .86$

The distance to the maximum ground level concentration is:

$$x_{max} = \left(\frac{qH^2}{(p+q)c_z^{\,2}}\right)^{\frac{1}{2}q}$$

$$x_{max} = 3.95 \cdot H^{1.162}$$

$$c_{max} = \frac{2Q(c_z/c_y)}{\pi \cdot e \cdot u \cdot H^2}$$

$$= .158 \frac{Q}{UH^2}$$

	PLANT I		PLANT II		
$Q(g/s)$	85		166		
$h_s(m)$	72		200		
	Holland	Briggs	Holland	Briggs	
$H(m)$	156	235	325	550	
$x_{max}(m)$	1360	2250	3275	6044	
U (m/s)	4.6	4.6	5.9	5.9	←($U = \frac{u_{100}}{3.16} h_s^{1/4}$
C_{max} ($\mu g/m^3$)	119	53	42	14	←(sulphur content in oil .3%).

As seen from this table, the maximum ground level concentration from plant II is 3 times as high using the Holland plume rise formula as it is using Briggs plume rise equation.

This demonstrates the importance of a proper choice of plume rise formula when estimating air quality.